Math 16A (Bernd Sturmfels), **Midterm Exam # 2**
Tuesday, November 8, 8:10 a.m.–9:25 a.m.

This exam is closed book. Please do not use any notes, calculators, cell phones etc. You must show all your all work to get credit. Write sentences if time permits. Each problem is worth 20 points, for a total of 100 points.

(1) A club offers memberships at the price of $400, provided a minimum of 150 people join. For each member in excess of 150, the membership fee will be reduced by $2 per person (for each member). How many memberships should the club sell in order to maximize its revenue ?

(2) Determine the derivatives of the following functions:
   (a) $(x^4 - 1)/(2x + 1)$
   (b) $2^x + 3^x \cdot 5^x$
   (c) $x^{\sqrt{x}}$
   (d) $\ln(3x) + \ln(5x) - \ln(17x^2)$

(3) In this problem we consider the composition of three functions.
   (a) Let $f(x), g(x)$ and $h(x)$ be differentiable functions. Derive a formula for the derivative of the function $f\big(g(h(x))\big)$.
   (b) Write the function $\ln\big(1 + \sqrt{7 + e^x}\big)$ in the form $f\big(g(h(x))\big)$. Compute the derivative of this function using your formula from (a).

(4) A point is moving along the circle $(x - 1)^2 + (y - 2)^2 = 13$. When the point is at $(4, 4)$, its x-coordinate is increasing at the rate of 4 units per second. Is the y-coordinate increasing or decreasing at that moment ? How fast is it increasing or decreasing ?

(5) In the study of epidemics, one finds the equation

$$\ln(1 - y) - \ln y = C - rt,$$

where y is the fraction of the population that has a specific disease at time t. Here the constants C and r are positive numbers.
   (a) Solve the equation for y in terms of t and the constants C and r.
   (b) According to this model, does the fraction of sick people in the population increase or decrease as t increases ?