1. Let $c$ denote the circumference of the ellipse defined by $2x^2 + 3y^2 = 5$. Compute $c$ numerically up to 15 digits of accuracy. Express $c$ as an integral $\int_\gamma f dg$ of a holomorphic differential on a Riemann surface.

2. Let $C$ be the quartic in $\mathbb{P}^2$ defined by $x^4 + y^4 = z^4$. Find a meromorphic differential on $C$ and compute the corresponding canonical divisor.

3. Find an irreducible homogeneous polynomial $P(x, y, z)$ of degree 6 such that the curve $\{P(x, y, z) = 0\}$ has precisely 10 singular points in $\mathbb{P}^2$.

4. Let $C$ be a smooth curve in complex projective 3-space $\mathbb{P}^3$ that is the intersection of two surfaces of degree $d$ and $e$. What is the genus of $C$?

5. [Kirwan 6.1] Show that the integral of an exact holomorphic differential along a closed piecewise-smooth path on a Riemann surface $S$ is 0. Deduce that the homologomorphic differential $\eta$ on $\mathbb{C}/\Lambda$ is not exact.

6. [Kirwan 6.15] Show that any two nonzero holomorphic differentials on a compact connected Riemann surface are constant multiples of each other.

7. True or false: If $D$ is a divisor on a nonsingular curve in $\mathbb{P}^2$ then the function $\mathbb{N} \to \mathbb{N}$, $m \mapsto l(mD)$ is given by a polynomial in $m$.

8. [Fulton 8.30] Given a nonsingular curve in $\mathbb{P}^2$, suppose that $D$ and $D'$ are divisors whose sum $D + D'$ is a canonical divisor. Then

$$l(D) - l(D') = \frac{1}{2}(\deg(D) - \deg(D')).$$