Math 255: Algebraic Curves

Bernd Sturmfels, UC Berkeley, Fall 2011
Homework # 2, due Tuesday, September 13

1. *[Kirwan 3.1]* Let C and D be curves in $\mathbb{P}^2$ with no common components. Show that $\mathrm{Sing}(C \cup D) = \mathrm{Sing}(C) \cup \mathrm{Sing}(D) \cup (C \cap D)$. Deduce that every reduced curve in $\mathbb{P}^2$ has only finitely many singular points.

2. *[Kirwan 3.3]* Show that any five points in $\mathbb{P}^2$ lie on a conic. Deduce that every curve of degree 4 in $\mathbb{P}^2$ with 4 singular points is reducible.

3. *[Kirwan 3.6]* State and prove Pappus' Theorem.

4. *[Kirwan 3.13]* State and prove the Cayley-Bacharach Theorem.

5. *[Kirwan 3.16]* Show that if p is an inflection point of a nonsingular cubic curve C in $\mathbb{P}^2$ then there are exactly four tangent lines to C which pass through p.

6. *[Kirwan 4.1]* Let C and D be nonsingular curves of degrees n and m in $\mathbb{P}^2$. Show that if C is homeomorphic to D then either $m = n$ or $\{n, m\} = \{1, 2\}$.

7. *[Kirwan 4.4]* Show that the map $(s : t : 0) \mapsto (st^3 : (s+t)^4 : t^4)$ defines a homeomorphism from the line $\{z = 0\}$ to a quartic curve in $\mathbb{P}^2$. Why does this not contradict the statement in the previous exercise?