1. [Kirwan 3.1] Let $C$ and $D$ be curves in $\mathbb{P}^2$ with no common components. Show that $\text{Sing}(C \cup D) = \text{Sing}(C) \cup \text{Sing}(D) \cup (C \cap D)$. Deduce that every reduced curve in $\mathbb{P}^2$ has only finitely many singular points.

2. [Kirwan 3.3] Show that any five points in $\mathbb{P}^2$ lie on a conic. Deduce that every curve of degree 4 in $\mathbb{P}^2$ with 4 singular points is reducible.


5. [Kirwan 3.16] Show that if $p$ is an inflection point of a nonsingular cubic curve $C$ in $\mathbb{P}^2$ then there are exactly four tangent lines to $C$ which pass through $p$.

6. [Kirwan 4.1] Let $C$ and $D$ be nonsingular curves of degrees $n$ and $m$ in $\mathbb{P}^2$. Show that if $C$ is homeomorphic to $D$ then either $m = n$ or $\{n, m\} = \{1, 2\}$.

7. [Kirwan 4.4] Show that the map $(s : t : 0) \mapsto (st^3 : (s+t)^4 : t^4)$ defines a homeomorphism from the line $\{z = 0\}$ to a quartic curve in $\mathbb{P}^2$. Why does this not contradict the statement in the previous exercise?