1. [Kirwan 2.2] Find the singular points, and the tangent lines at the singular points, of each of the following curves in \( \mathbb{C}^2 \):
   (a) \( y^3 - y^2 + x^3 - x^2 + 3y^2x + 3x^2y + 2xy = 0 \).
   (b) \( x^4 + y^4 - x^2y^2 = 0 \).
   (c) \( y^2 = x^3 - x \).

2. [Kirwan 2.4] Let \((a, b)\) be a singular point of an affine curve \( C \) defined by a polynomial \( P(x, y) \). Show that \((a, b)\) is an ordinary double point if and only if, at the point \((a, b)\),
   \[
   \left( \frac{\partial^2 P}{\partial x \partial y} \right)^2 \neq \left( \frac{\partial^2 P}{\partial x^2} \right) \left( \frac{\partial^2 P}{\partial y^2} \right).
   \]

3. [Kirwan 2.5] Let \( C \) be an affine curve defined by a polynomial \( P(x, y) \) of degree \( d \). Show that if \((a, b)\) is a point of multiplicity \( d \) in \( C \) then \( P(x, y) \) is a product of \( d \) linear factors, so \( C \) is the union of \( d \) lines through \((a, b)\).

4. [Kirwan 2.7] Show that a complex algebraic curve in \( \mathbb{C}^2 \) is never compact.

5. [Kirwan 2.9] For which values of \( \lambda \in \mathbb{C} \) are the following projective curves in \( \mathbb{P}^2 \) nonsingular? Describe the singularities when they exist:
   (a) \( x^3 + y^3 + z^3 + \lambda xyz = 0 \).
   (b) \( x^3 + y^3 + z^3 + \lambda(x + y + z)^3 = 0 \).
6. [Kirwan 2.8] The multiplicity of a point \((a : b : c)\) of a projective curve 
\(P(x, y, z) = 0\) is the smallest integer \(m\) such that

\[
\frac{\partial^m P}{\partial x^i \partial y^j \partial z^k} (a, b, c) \neq 0
\]

for some \(i, j, k\) such that \(i + j + k = m\). Find the singular points and
their multiplicities for the following projective curves:

(a) \(xy^4 + yz^4 + xz^4 = 0\).
(b) \(x^2y^3 + x^2z^3 + y^2z^3 = 0\).
(c) \(y^2z = x(x - z)(x - \lambda z) = 0\) for \(\lambda \in \mathbb{C}\).
(d) \(x^n + y^n + z^n = 0\) for any integer \(n > 0\).