Math 239/Stat 260: Algebraic Statistics

Homework # 4
Due: Tuesday, October 14, 2008

1. Consider the model for four binary random variables specified by the following set of three conditional independence statements
$$\mathcal{C} \;=\; \big\{ 1 \perp\!\!\!\perp 4 \,|\, \{2,3\}\,,\; 2 \perp\!\!\!\perp 3 \,|\, 4\,,\; 3 \perp\!\!\!\perp \{2,4\} \big\}.$$
   - Determine the polynomial ideal $I_{\mathcal{C}}$ representing this conditional independence model. What is the dimension of this model ?
   - Compute the primary decomposition of the ideal $I_{\mathcal{C}}$. Discuss the statistical meaning of the irreducible components.
   - Find an directed acyclic graph (DAG) for which precisely these three CI statements represent the local directed Markov property.
   - Verify the directed Hammersley-Clifford Theorem (Theorem 3.2.10) for your DAG.

2. Prove the following cyclic implication among conditional independence statements for Gaussian random variables: If $1 \perp\!\!\!\perp 2 \,|\, 3$, $2 \perp\!\!\!\perp 3 \,|\, 4$, $3 \perp\!\!\!\perp 4 \,|\, 5$, $4 \perp\!\!\!\perp 5 \,|\, 1$ and $5 \perp\!\!\!\perp 1 \,|\, 2$ then $1 \perp\!\!\!\perp 2$, $2 \perp\!\!\!\perp 3$, $3 \perp\!\!\!\perp 4$, $4 \perp\!\!\!\perp 5$ and $5 \perp\!\!\!\perp 1$. Does the same implication hold for discrete random variables?

3. Let G be the (undirected) edge graph of the 3-dimensional cube. Determine all conditional independence statements for the undirected pairwise Markov property on G, and also for the undirected global Markov property on G. Does there exist a probability distribution on eight binary random variables that satisfies the former but not the latter?

4. Make a serious attempt at solving the first problem in Section 7.10 (page 163) in the case when X_1, X_2, X_4 are binary and X_3 is ternary. What is your opinion about the two questions in Section 7.11 ? Do you think that the answer is “yes” or do you think that the answer is “no”?