

Math 239/Stat 260: Algebraic Statistics

Homework # 2

Due: Tuesday, September 23, 2008

1. Consider the independence model for two random variables on the state space $\{\mathbf{A}, \mathbf{C}, \mathbf{G}, \mathbf{T}\}$. We are given a sample of size 40 with table of counts

$$U = \begin{pmatrix} 4 & 2 & 2 & 2 \\ 2 & 4 & 2 & 2 \\ 2 & 2 & 4 & 2 \\ 2 & 2 & 2 & 4 \end{pmatrix}$$

Compute the *conditional p-value* (1.1.12), either exactly in rational arithmetic, or numerically using Metropolis-Hastings and Corollary 1.15.

2. Consider the model of complete independence for six binary random variables that are identically distributed. Let A be the corresponding matrix with distinct columns. Compute a lattice basis, the Markov bases, a Gröbner basis, and the Graver basis of the lattice $\mathcal{L} = \text{kernel}(A)$.
3. Find an example of a hierarchical model whose minimal Markov basis contains a binomial $p^u - p^v$ of odd degree.
4. What is the ML degree of the model in Question 2? Can you give a formula for the maximum likelihood estimate in terms of the data?
5. This concerns Problem 7.2 of the course text. Feel free to consult Alex.
 - Explain in your own words why this problem might be interesting.
 - Determine the Markov width of the cycle of length five.
6. Implement Iterative Proportional Scaling (Algorithm 2.1.9), and test your code on the log-linear models in Questions 1 and 2, and on the No-Three-Way interaction model in Example 2.1.8. What do you observe?