Fitness at Fields: **Surfaces**

Wednesday, August 24, 2016

1. A nondegenerate surface in $\mathbb{P}^n$ has degree at least $n-1$. Prove this fact and determine all surfaces of degree $n-1$. Give their equations.

2. How many lines lie on a surface obtained by intersecting two quadratic hypersurfaces in $\mathbb{P}^4$? Find an instance where all lines are defined over $\mathbb{Q}$.

3. What is the maximum number of singular points on an irreducible quartic surface in $\mathbb{P}^3$? Find a surface and compute its *projective dual*.

4. Given a general surface of degree d in $\mathbb{P}^3$, the set of its *bitangent lines* is a surface in $\mathrm{Gr}(1, \mathbb{P}^3)$. Determine the bidegree of that surface.

5. Pick two random circles C_1 and C_2 in $\mathbb{R}^3$. Compute their *Minkowski sum* $C_1 + C_2$ and their *Hadamard product* $C_1 \star C_2$. Try other curves.

6. Let X be the surface obtained by blowing up 5 points in the plane. Compute the *Cox ring* of X. Which of its ideals describe points on X?

7. The incidences among the 27 lines on a cubic surface defines a 10-regular graph. Compute the complex of independent sets in this graph.

8. The Hilbert scheme of points on a smooth surface is smooth. Why? How many torus-fixed points are there on the Hilbert scheme of 20 points in $\mathbb{P}^2$? What can you say about the graph that connects them?

9. State the *Hodge Index Theorem*. Verify this theorem for cubic surfaces in $\mathbb{P}^3$, by explicitly computing the matrix for the intersection pairing.

10. List the equations of one *Enriques surface*. Verify its Hodge diamond.