Fitness at Fields: **Parameter and Moduli**

Friday, September 2, 2016

1. Write down (in `M2` format) the two generators of the *ring of invariants* for ternary cubics. For which plane cubics do both invariants vanish?

2. Fix a $\mathbb{Z}$-grading on the polynomial ring $S = \mathbb{C}[a, b, c, d]$ by $\deg(a) = 1$, $\deg(b) = 4$, $\deg(c) = 5$, and $\deg(d) = 9$. Classify all homogeneous ideals I such that S/I has Hilbert function identically equal to 1.

3. Consider the Hilbert scheme of eight points in affine 4-space $\mathbb{A}^4$. Identify a point that is not in the main component. List its ideal generators.

4. Let X be the set of all symmetric 4×4-matrices in $\mathbb{R}^{4\times 4}$ that have an eigenvalue of multiplicity ≥ 2. Compute the $\mathbb{C}$-Zariski closure of X.

5. Which cubic surfaces in $\mathbb{P}^3$ are stable? Which ones are semi-stable?

6. In his second lecture on August 15, Valery Alexeev used six lines in $\mathbb{P}^2$ to construct a certain moduli space of K3 surfaces with 15 singular points. List the most degenerate points in the boundary of that space.

7. Find the most singular point on the Hilbert scheme of 16 points in $\mathbb{A}^3$.

8. The polynomial ring $\mathbb{C}[x, y]$ is graded by the 2-element group $\mathbb{Z}/2\mathbb{Z}$ via $\deg(x)$=$\deg(y)$=1. Classify all Hilbert functions of homogeneous ideals.

9. Consider all threefolds gotten by blowing up six points in $\mathbb{P}^3$. Describe their Cox rings and Cox ideals. How to compactify the moduli space?

10. The moduli space of tropical curves of genus 5 is a polyhedral space of dimension 12. Determine the number of i-faces for $i = 0, 1, 2, \ldots, 12$.

Bonus question: Which is the earliest paper in *combinatorial algebraic geometry*?