Fitness at Fields: **Grassmannians**

Friday, August 26, 2016

1. Find a point in $\mathrm{Gr}(3,6)$ with precisely 16 non-zero Plücker coordinates. As in June Huh's intro course, determine the Chow ring of its *matroid*.

2. The coordinate ring of the Grassmannian $\mathrm{Gr}(3,6)$ is a *cluster algebra* of finite type. What are the cluster variables? List all the clusters.

3. Consider two general surfaces in $\mathbb{P}^3$ whose degrees are d and e respectively. How many lines in $\mathbb{P}^3$ are *bitangent* to both surfaces?

4. The *rotation group* $\mathrm{SO}(n)$ is an affine variety in the space of real $n \times n$-matrices. Can you find a formula for the degree of this variety?

5. The *complete flag variety* for $\mathrm{GL}(4)$ is a six-dimensional subvariety of $\mathbb{P}^3 \times \mathbb{P}^5 \times \mathbb{P}^3$. Compute its ideal and determine its tropicalization.

6. Classify all toric ideals that arises as initial ideals for flag variety above. For each such toric degeneration, compute the *Newton-Okounkov body*.

7. The Grassmannian $\mathrm{Gr}(4,7)$ has dimension 12. Four *Schubert cycles* of codimension 3 intersect in a finite number of points. How large can that number be? Exhibit explicit cycles whose intersection is reduced.

8. The *affine Grassmannian* and the *Sato Grassmannian* are two infinite-dimensional versions of the Grassmannian. How are they related?

9. The coordinate ring of the Grassmannian $\mathrm{Gr}(2,7)$ is $\mathbb{Z}^7$-graded. Determine the Hilbert series and the multidegree of $\mathrm{Gr}(2,7)$ for this grading.

10. The *Lagrangian Grassmannian* parametrizes n-dimensional isotropic subspaces in $\mathbb{C}^{2n}$. Find a Gröbner basis for its ideal. What is a "doset"?