Fitness at Fields: **Curves**

Monday, August 22, 2016

1. Which genus can a smooth curve of degree 6 in $\mathbb{P}^3$ have? Give examples.
2. Let $f(x) = (x-1)(x-2)(x-3)(x-6)(x-7)(x-8)$ and consider the genus 2 curve $y^2 = f(x)$. Where is it in $\mathcal{M}_2$? Compute the Igusa invariants. Draw the Berkovich skeleton for the field of 5-adic numbers.
3. The *tact invariant* of two plane conics is the polynomial of bidegree $(6,6)$ in the $6+6$ coefficients which vanishes when the conics are tangent. Compute this invariant explicitly. How many terms does it have?
4. *Bring's curve* in $\mathbb{P}^3$ is defined by $x_0^i + x_1^i + x_2^i + x_3^i + x_4^i = 0$ for $i = 1, 2, 3$. What is its genus? Determine all tritangent planes of this curve.
5. Let X be a curve of degree d and genus g in $\mathbb{P}^3$. The Chow form of X defines a hypersurface in the Grassmannian $\mathrm{Gr}(1, \mathbb{P}^3)$. Points are lines that meet X. Find the dimension and (bi)degree of its singular locus.
6. What are the equations of the secant varieties of *elliptic normal curves*?
7. Let X_P be the *toric variety* defined by a 3-dimensional lattice polytope, as in Milena's July 18-22 course. Intersect X_P with two general hyperplanes to get a curve. What is the degree and genus of that curve?
8. A 2009 article by Sean Keel and Jenia Tevelev presents *Equations for* $\overline{M}_{0,n}$. Write these equations in `Macaulay2` format for $n = 5$ and $n = 6$. Can you see the psi-classes [Cavalieri, Section 2.5] in these coordinates?
9. Review the statement of *Torelli's Theorem* for genus 3. Using `sage` or `maple`, compute the 3×3 Riemann matrix of the Fermat quartic $\{x^4 + y^4 + z^4 = 0\}$. How can you recover the curve from that matrix?
10. The moduli space $\mathcal{M}_7$ of genus 7 curves has dimension 18. What is the codimension of the locus of plane curves? (Singularities are allowed).