Fitness at Fields: **Convexity**

Monday, August 29, 2016

1. The set of nonnegative binary sextics is a closed full-dimensional convex cone in $\mathrm{Sym}_6(\mathbb{R}^2) \simeq \mathbb{R}^7$. Determine the face poset of this convex cone.

2. Consider smooth projective toric fourfolds with 8 invariant divisors. What is the maximal number of torus-fixed points of any such variety?

3. Choose three general ellipsoids in $\mathbb{R}^3$ and compute the convex hull of their union. Which algebraic surfaces contribute to the boundary?

4. Explain how the Alexandrov-Fenchel Inequalities (for Convex Bodies) can be derived from the Hodge Index Theorem (for Algebraic Surfaces).

5. The blow-up of $\mathbb{P}^3$ at six general points is a threefold that contains 32 special surfaces. What are these surfaces? Which triples intersect? Hint: Find a 6-dimensional polytope that describes the combinatorics.

6. Prove that every face of a spectrahedron is an exposed face.

7. How many combinatorial types of reflexive polytopes are there in dimension 3? In dimension 4? Draw pictures of some extreme specimen.

8. A 4×4-matrix has six off-diagonal 2×2-minors. Their binomial ideal in 12 variables has a unique toric components. Determine the f-vector of the polytope (with 12 vertices) associated with this toric variety.

9. Consider the Plücker embedding of the real Grassmannian $\mathrm{G}(2, 5)$ in the unit sphere in $\mathbb{R}^{10}$. Describe its convex hull. (Hint: Calibrations, Orbitopes).

10. Examine Minkowski sums of three tetrahedra in $\mathbb{R}^3$. What is the maximum number of vertices such a polytope can have? How to generalize?