Fitness at Fields: **Abelian Combinatorics**

Wednesday, August 31, 2016

1. The intersection of two quadratic surfaces in $\mathbb{P}^3$ is an *elliptic curve*. Explain its group structure in terms of geometric operations in $\mathbb{P}^3$.

2. A 2006 paper by Keiichi Gunji gives explicit equations for all *abelian surfaces* in $\mathbb{P}^8$. Verify his equations in `M2`. How to find the group law?

3. Experiment with Swierczewski's `Sage` code for the numerical evaluation of the *Riemann theta function* $\theta(\tau; z)$. Verify the functional equation.

4. *Theta functions with characteristics* $\theta[\epsilon, \epsilon'](\tau; z)$ are indexed by two binary vectors $\epsilon, \epsilon' \in \{0, 1\}^g$. They are odd or even. How many each?

5. Fix the symplectic form $\langle x, y\rangle = x_1y_4+x_2y_5+x_3y_6+x_4y_1+x_5y_2+x_6y_3$ on the 64-element vector space $(\mathbb{F}_2)^6$. Determine all isotropic subspaces.

6. Explain the combinatorics of the root system of type E_7. How would you choose coordinates? How many pairs of roots are orthogonal?

7. In 1879 Cayley published a paper in Crelle's journal titled *Algorithms for ...* What did he do? How does it relate the previous two exercises?

8. The *regular matroid* R_{10} defines a degeneration of abelian 5-folds. Describe its periodic tiling on $\mathbb{R}^5$ and secondary cone in the 2nd Voronoi decomposition. Explain the application to Prym varieties due to Gwena.

9. Consider the Jacobian of the plane quartic curve defined over $\mathbb{Q}_2$ by
$$41x^4 + 1530x^3y + 3508x^3z + 1424x^2y^2 + 2490x^2yz - 2274x^2z^2 + 470xy^3$$
$$+680xy^2z-930xyz^2+772xz^3+535y^4-350y^3z-1960y^2z^2-3090yz^3-2047z^4$$
Compute its limit in *Alexeev's moduli space* for the 2-adic valuation.

10. Let Θ be the *theta divisor* on an abelian 3fold X. Find $n = \dim H^0(X, k\Theta)$. What is the smallest integer k such that $k\Theta$ very ample? Can you compute (in `M2`) the ideal of the corresponding embedding $X \hookrightarrow \mathbb{P}^{n-1}$?