Math 113, **First Midterm Exam**
Thursday, February 18, 10:15am–11:45am

This exam is open book. You may use the text book (to which you can refer) and your own notes, but no electronic devices. Please write your answers in a blue note book. There are five problems, each worth 10 *points, for a total of* 50 *points. Answers without justification will not receive credit. You may look at your graded exam in class on Monday, July 12.*

(1) Let G be a group of order pq where p and q are prime numbers. Show that every proper subgroup of G is cyclic. Is the group G necessarily abelian?

(2) Compile a list of all subgroups of the alternating group A_4 and draw a subgroup diagram.

(3) How many distinct homomorphims are there from the additive group of integers $\mathbf{Z}$ to the cyclic group $\mathbf{Z}_{20}$? How many of them are injective? How many are surjective?

(4) The dihedral group D_5 consists of all symmetries of a regular pentagon. Find its commutator subgroup C, and determine the factor group D_5/C.

(5) Let G be a group and consider the set $H = \{(g, g) \mid g \in G\}$.
  (a) Show that H is a subgroup of $G{\times}G$.
  (b) Show that H is a normal subgroup of $G{\times}G$ if and only if G is abelian.
  (c) Assuming that G is abelian, show that $(G{\times}G)/H$ is isomorphic to G.