QUALIFYING EXAM SYLLABUS

BENSON AU

COMMITTEE: DAVID ALDOUS (CHAIR), STEVEN EVANS, DMITRY LIVDAN (HAAS), DAN-VIRGIL VOICULESCU.

1. Major Topic: Probability Theory (Probability)

Processes, Distributions, and Independence: Random elements and processes; distributions and expectation; independence; zero-one laws; Borel-Cantelli lemma. (Durrett: 2.1, 2.3; Kallenberg: 3)

Random Sequences, Series, and Averages: Convergence in probability and in L^p; uniform integrability and tightness; convergence in distribution; convergence of random series; strong laws of large numbers; Portmanteau theorem. (Durrett: 2.4, 2.5; Kallenberg: 4)

Characteristic Functions and Classical Limit Theorems: Uniqueness and continuity theorem; Poisson convergence; positive and symmetric terms; Lindeberg's condition; general Gaussian convergence; weak laws of large numbers; vague and weak compactness. (Durrett: 2.2, 3.1-3.3, 3.4, 3.6, 3.9; Kallenberg: 5)

Conditioning and Disintegration: Conditional expectations and probabilities; regular conditional distributions; disintegration; conditional independence. (Durrett: 5.1; Kallenberg: 6)

Martingales and Optional Times: Filtrations and optional times; martingale property; optional stopping and sampling; maximum and upcrossing inequalities; martingale convergence; limits of conditional expectations. (Durrett: 5.2-5.7; Kallenberg: 7)

Markov Chains: Extensions of the Markov property; recurrence and transience; stationary measures; asymptotic behavior. (Durrett: 6.1-6.6)

Stationary Processes and Ergodic Theory: Stationarity, invariance, and ergodicity; discrete-time ergodic theorems; subadditivity and products of random matrices. (Durrett: 7.1-7.5; Kallenberg: 10)

Gaussian Processes and Brownian Motion: Symmetries of Gaussian distributions; existence and path properties of Brownian motion; strong Markov and reflection properties; arcsine and uniform laws. (Durrett: 8.1-8.4; Kallenberg: 13)

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2. Major Topic: Von Neumann Algebras (Analysis)

Definition and Basic Properties: Topologies on $\mathcal{B}(\mathfrak{H})$; predual; Borel functional calculus; von Neumann bicommutant theorem; abelian von Neumann algebras; Kaplansky density theorem. (Jones: 2, 3, 5, 8)

Projections: Comparison of projections; Murray-von Neumann equivalence; Schröder-Bernstein theorem; reduced and induced von Neumann algebras. (Jones: 6)

Factors: Factors of type I, II_1, II_∞, and III; traces; standard form of a II_1-factor; hyperfiniteness; coupling constant. (Jones: 4, 6, 7, 9, 10, 15)

3. Minor Topic: Random Matrix Theory (Probability)

Methodologies and Classical Distributions: Traces, moments, and combinatorics; Stieltjes transforms and recursions; the semicircular law; maximal eigenvalues and Füredi-Komlós enumeration; the Marčenko-Pastur law. (AGZ: 2.1.1-2.1.6, 2.2, 2.4; Tao: 2.4.1-2.4.3)

Gaussian Ensembles: Joint distribution of eigenvalues in the GOE and the GUE; Hermite polynomials and the GUE; tridiagonal form of the GUE. (AGZ: 2.5.1-2.5.3, 3.2, 3.3.1; Tao: 2.6.5)

References

1. Greg W. Anderson, Alice Guionnet, and Ofer Zeitouni. *An introduction to random matrices*, volume 118 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 2010.
2. Rick Durrett. *Probability: theory and examples.* Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, Cambridge, fourth edition, 2010.
3. Vaughan Jones. *Von Neumann Algebras.* `http://math.berkeley.edu/~vfr/MATH20909/VonNeumann2009.pdf`
4. Olav Kallenberg. *Foundations of modern probability.* Probability and its Applications (New York). Springer-Verlag, New York, second edition, 2002.
5. Terence Tao. *Topics in random matrix theory*, volume 132 of *Graduate Studies in Mathematics.* American Mathematical Society, Providence, RI, 2012.