TRUE/FALSE:

1. Any set of 5 vectors in $\mathbb{R}^4$ is linearly dependent.
2. Any set of 5 vectors in $\mathbb{R}^4$ spans $\mathbb{R}^4$.
3. A basis for $\mathbb{R}^4$ always consists of 4 vectors.
4. The union of two subspaces is a subspace.
5. There exists a subspace of $\mathbb{R}^2$ containing exactly 2 vectors.
6. There exists a subspace of $\mathbb{R}^2$ containing exactly 1 vector.
7. If $A : \mathbb{R}^5 \to \mathbb{R}^3$ is a linear transformation, the dimension of the null space of A is at least 2.
8. A set of three vectors is linearly dependent only if one of them is a scalar multiple of another.
9. Row operations do not change the null space of a matrix.
10. Row operations do not change the column space of a matrix.
11. Row operations do not change the determinant of a matrix.
12. If the determinant of a $5x5$ matrix A is equal to -7, then A is invertible.
13. If the determinant of a $3x3$ matrix A is equal to 0, then the dimension of Col A is at most 2.
14. det $(A + B) =$ det $A +$ det B for any two square matrices A and B of the same size.
15. det $(-A) = -$det A for any square matrix A.

CALCULATIONS:

1. Find a basis for Col A and Nul A. Find the dimensions of each subspace, and verify that the rank-nullity theorem holds.

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 4 & 12 & -8 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & 6 & -3 & 7 \\ -1 & -3 & 6 & -8 \end{bmatrix} \qquad A = \begin{bmatrix} 5 & -1 & -2 \\ -1 & 5 & -2 \\ -2 & -2 & 2 \end{bmatrix}$$

2. Let $v = [1, 2, 3]^T$. For each of the following matrices A, determine the following: Is A one-to-one? Is A onto? Is A invertible? Is v in Col A? Is v in Nul A? What is the determinant of A? If A is invertible, what is the inverse of A?

$$A = \begin{bmatrix} 2 & -4 & 2 \\ 1 & 4 & -3 \\ 0 & -9 & 6 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 7 & 20 \\ 0 & 4 & 3 \\ 0 & 2 & 1 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 5 & 13 \\ -2 & -2 & -4 \\ 1 & 5 & 14 \end{bmatrix}$$