Quiz #2: 2/2
MATH 110
Section 119: 10:10 - 11:00
GSI: Alex Zorn

Name: ________________________________

You have 15 minutes to complete this quiz. For full credit you must explain all your reasoning.

1. (5 pts) Is the set $W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_{2\times 2}(\mathbb{R}) \mid ad - bc = 0 \right\}$ a subspace of $M_{2\times 2}(\mathbb{R})$? Justify your answer.

Solution: It is not. For example, $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \in W$, and $B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \in W$, but $A+B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is not in W. So W is not closed under addition.

2. (5 pts) Let $p_1 = 1$, $p_2 = x^2$, $p_3 = x^3 - x$ be elements of $P_3(\mathbb{R})$. Show that the span of $\{p_1, p_2, p_3\}$ is the set of all polynomials p in $P_3(\mathbb{R})$ satisfying $p(1) = p(-1)$.

Solution: Let $V = \text{span}(\{p_1, p_2, p_3\})$, and W be the set of of all polynomials p in $P_3(\mathbb{R})$ satisfying $p(1) = p(-1)$. Need to show:

$\mathbf{V} \subseteq \mathbf{W}$:

Let $p \in V$. Then $\vec{x} = ap_1 + bp_2 + cp_3$ for some $a, b, c \in \mathbb{R}$. So $p(1) = ap_1(1) + bp_2(1) + cp_3(1) = a + b$, and $p(-1) = ap_1(-1) + bp_2(-1) + cp_3(-1) = a + b$. So, $p \in W$.

$\mathbf{W} \subseteq \mathbf{V}$:

Let $p \in W$, with $p = a + bx + cx^2 + dx^3$. Then $p(1) = a + b + c + d$ and $p(-1) = a - b + c - d$. So:

$$a + b + c + d = a - b + c - d$$

$$\Rightarrow 2b + 2d = 0$$

$$\Rightarrow d = -b$$

$$\Rightarrow p = a + bx + cx^2 + (-b)x^3 = a + cx^2 + b(x - x^3) = ap_1 + cp_2 + (-b)p_3$$

So $p \in V$.