Quiz #2: 2/2
MATH 110
Section 118: 9:10 - 10:00
GSI: Alex Zorn

Name: ________________________________

You have 15 minutes to complete this quiz. For full credit you must explain all your reasoning.

1. (5 pts) Is the set $W = \{f \in \mathcal{F}(\mathbb{R}, \mathbb{R}) \mid f(1) \cdot f(-1) = 0\}$ a subspace of $\mathcal{F}(\mathbb{R}, \mathbb{R})$? Justify your answer.

Solution: It is not. For example, $f(x) = 1 + x \in W$ since $f(1) \cdot f(-1) = 2 \cdot 0 = 0$, and $g(x) = 1 - x \in W$ since $g(1) \cdot g(-1) = 0 \cdot 2 = 0$ but $(f + g)(x) = (1 + x) + (1 - x) = 2$ is not in W. So W is not closed under addition.

2. (5 pts) Let $S = \{(1,-1,0),(0,1,-1),(1,0,-1)\}$. Show that the span of S equals $\{(x,y,z) \in \mathbb{R}^3 \mid x+y+z=0\}$.

Solution: Let $V = \text{span}(S)$, and $W = \{(x,y,z) \in \mathbb{R}^3 \mid x+y+z=0\}$. Need to show:

$\mathbf{V} \subseteq \mathbf{W}$:

Let $\vec{x} = (x,y,z) \in V$. Then $\vec{x} = a(1,-1,0) + b(0,1,-1) + c(1,0,-1) = (a+c, -a+b, -b-c)$ for some $a,b,c \in \mathbb{R}$. So $x+y+z = (a+c)+(-a+b)+(-b-c) = 0$, so $\vec{x} \in W$.

$\mathbf{W} \subseteq \mathbf{V}$:

Let $\vec{x} = (x,y,z) \in W$. Then $x+y+z=0$, so $z = -x-y$. We want to find a,b,c such that $a(1,-1,0)+b(0,1,-1)+c(1,0,-1) = (a+c,-a+b,-b-c)$

$$a + c = x$$

$$-a + b = y$$

$$-b - c = -x - y$$

This system has a solution:

$$a = 0$$

$$b = y$$

$$c = x$$

(This is not the only solution, but it is a solution). So $\vec{x} \in W$.