Quiz #2: 2/2
MATH 110
Section 101: 8:10 - 9:00
GSI: Alex Zorn

Name: ________________________________

You have 15 minutes to complete this quiz. For full credit you must explain all your reasoning.

1. (5 pts) Is the set $W = \{(x, y, z) \in \mathbb{R}^3 \mid xyz = 0\}$ a subspace of $\mathbb{R}^3$? Justify your answer.

Solution: It is not. For example, $(1, 0, 0) \in W$ and $(0, 1, 1) \in W$, but $(1, 0, 0) + (0, 1, 1) = (1, 1, 1) \notin W$, so W is not closed under addition.

2. (5 pts) The **trace** of a square matrix is defined to be the sum of its diagonal entries. Show that if

$$M_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}; \qquad M_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}; \qquad M_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Then the span of $\{M_1, M_2, M_3\}$ is the set of all 2×2 matrices whose trace equals zero.

Solution: Let $V = \text{span}(\{M_1, M_2, M_3\})$, and W the set of all 2×2 matrices whose trace equals zero. Need to show:

$\mathbf{V} \subseteq \mathbf{W}$:

Let $A \in V$. Then $A = aM_1 + bM_2 + cM_3$ for some $a, b, c \in \mathbb{R}$. So $A = \begin{bmatrix} c & a+b \\ a-b & -c \end{bmatrix} \Rightarrow \text{tr}(A) = c - c = 0 \Rightarrow A \in W$.

$\mathbf{W} \subseteq \mathbf{V}$:

Let $A \in W$. Then $\text{tr}(A) = 0$, so $A = \begin{bmatrix} x & y \\ z & -x \end{bmatrix}$ for some $x, y, z \in \mathbb{R}$. We want to find a, b, c such that $A = aM_1 + bM_2 + cM_3 = \begin{bmatrix} c & a+b \\ a-b & -c \end{bmatrix}$. So:

$$c = x$$

$$a + b = y$$

$$a - b = z$$

$$-c = -c$$

This system has the solution:

$$a = \frac{y+z}{2}$$

$$b = \frac{y-z}{2}$$

$$c = x$$

So $A \in V$.