Quiz #1: 1/26
MATH 110
Section 119: 10:10 - 11:00
GSI: Alex Zorn

Name: ________________________________

You have 15 minutes to complete this quiz. For full credit you must explain all your reasoning.

1. (5 pts) Let V denote the set of differentiable real-valued functions f on the real line satisfying $f'(x) = f(x)$. Prove that V is a vector space with the usual operations of addition and scalar multiplication for functions. (You may use any facts about differentiation from calculus without proof).

Solution: The set of all real-valued functions defined on the real line is a vector space. V is a subset. Only need to show:

-Closure under addition. If f and g are in V, then:

$$(f+g)'(x) = f'(x) + g'(x) = f(x) + g(x) = (f+g)(x)$$

So $f+g$ is in V.

-Closure under scalar multiplication. If f is in V and $c \in \mathbb{R}$, then:

$$(cf)'(x) = c(f'(x)) = cf(x) = (cf)(x)$$

So cf is in V.

-The set contains the zero function. Let f_0 denote the zero function. Then $f_0'(x) = 0 = f_0(x)$, so f_0 is in V.

2. (5 pts) Let V denote the set of ordered pairs of real numbers. If (x_1, y_1) and (x_2, y_2) are elements of V, and $c \in \mathbb{R}$, define:

$$(x_1, y_1) + (x_2, y_2) = (x_1 x_2, y_1 y_2) \qquad \text{and} \qquad c(x_1, y_1) = (cx_1, cy_1)$$

Show that V is not a vector space over $\mathbb{R}$ with these operations.

Solution: This construction does not satisfy VS8- for all $a, b \in \mathbb{R}$ and $x \in V$, $(a+b)x = ax + bx$. Because, for example:

$$(1+1)(1,1) = 2(1,1) = (2,2)$$
$$1(1,1) + 1(1,1) = (1,1) + (1,1) = (1,1)$$