Quiz #1 Solutions: 1/26
MATH 110
Section 118: 9:10 - 10:00
GSI: Alex Zorn

Name: ________________________________

You have 15 minutes to complete this quiz. For full credit you must explain all your reasoning.

1. (5 pts) Let V denote the set of real-valued functions f on the real line satisfying $f(3) = 0$. Prove that V is a vector space with the usual operations of addition and scalar multiplication for functions.

Solution: The set of all real-valued functions defined on the real line is a vector space. V is a subset. Only need to show:

-Closure under addition. If f and g are in V, then:

$$(f + g)(3) = f(3) + g(3) = 0 + 0 = 0$$

So $f + g$ is in V.

-Closure under scalar multiplication. If f is in V and $c \in \mathbb{R}$, then:

$$(cf)(3) = c(f(3)) = c \cdot 0 = 0$$

So cf is in V.

-The set contains the zero function. Let f_0 denote the zero function. Then $f_0(3) = 0$, so f_0 is in V.

2. (5 pts) Let V denote the set of ordered pairs of real numbers. If (x_1, y_1) and (x_2, y_2) are elements of V, and $c \in \mathbb{R}$, define:

$$(x_1, y_1) + (x_2, y_2) = (x_1^2 + x_2^2, y_1^2 + y_2^2) \qquad \text{and} \qquad c(x_1, y_1) = (cx_1, cy_1)$$

Show that V is not a vector space over $\mathbb{R}$ with these operations.

Solution: This construction does not satisfy VS7- for all $a, b \in \mathbb{R}$ and $x \in V$, $(a + b)x = ax + bx$. Because, for example:

$$(2 + 2)(1, 1) = 4(1, 1) = (4, 4)$$
$$2(1, 1) + 2(1, 1) = (2, 2) + (2, 2) = (2^2 + 2^2, 2^2 + 2^2) = (8, 8)$$