Quiz #1 Solutions: 1/26
MATH 110
Section 101: 8:10 - 9:00
GSI: Alex Zorn

Name: ________________________________

You have 15 minutes to complete this quiz. For full credit you must explain all your reasoning.

1. (5 pts) A real-valued function f defined on the real line is an **odd function** if $f(-t) = -f(t)$ for each real number t. Prove that the set of odd functions defined on the real line with the usual operations of addition and scalar multiplication for functions is a vector space.

Solution: The set of all real-valued functions defined on the real line is a vector space. This is a subset. Only need to show:

-Closure under addition. If f and g are odd functions, then:

$$(f+g)(-t) = f(-t) + g(-t) = -f(t) - g(t) = -(f+g)(t)$$

So $f+g$ is and odd function.

-Closure under scalar multiplication. If f is an odd function and $c \in \mathbb{R}$, then:

$$(cf)(-t) = c(f(-t)) = c(-f(t)) = -cf(t) = -(cf)(t)$$

So cf is an odd function.

-The set contains the zero function. Let f_0 denote the zero function. For any t, $f_0(-t) = 0$, $-f_0(t) = -0 = 0$. So f_0 is an odd function.

2. (5 pts) Let V denote the set of ordered pairs of real numbers. If (x_1, y_1) and (x_2, y_2) are elements of V, and $c \in \mathbb{R}$, define:

$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2) \qquad \text{and} \qquad c(x_1, y_1) = (cy_1, cx_1)$$

Show that V is not a vector space over $\mathbb{R}$ with these operations.

Solution: This construction does not satisfy VS5- for all $x \in V$, $1x = x$. Because, for example:

$$1(0, 1) = (1, 0) \neq (0, 1)$$