Practice midterm 3

Complex analysis, lecture 4

November 5, 2025

As usual, be sure to include your method, and remember to write your name.

Problem 1. Find the Taylor expansion of $f(z) = e^{2z} - e^{-2}$ centered at $z_0 = -1$. What is its radius of convergence?

The above problem is directed towards Objective 8 (power series).

Problem 2. Find the Taylor expansion of $f(z) = \frac{z^2}{z^3-1}$ at infinity.

The above problem is directed towards Objective 8 (power series).

Problem 3. Find the Laurent expansion of $f(z) = \frac{1}{(z+1)(z+2)}$ on the annulus $\{1 < |z| < 2\}$.

The above problem is directed towards Objective 9 (Laurent series).

Problem 4. Find and classify the isolated singularities of $f(z) = \frac{\cos(1/z)}{z^4-1}$. For any poles, find their order.

The above problem is directed towards Objective 9 (Laurent series).

Since we have had less time with the residue theorem, I've included an extra practice problem on it below (so three instead of two). However, only two will appear on the actual exam.

Problem 5. Let D be the rectangle in $\mathbb{C}$ with corners at i, $-i$, $4+i$, and $4-i$. Using the residue theorem, compute

$$\int_{\partial D} \frac{z}{\sin z}\, dz.$$

The above problem is directed towards Objective 10 (the residue theorem).

Problem 6. Using the residue theorem, compute

$$\int_{-\infty}^{\infty} \frac{1}{x^4+1}\, dx.$$

The above problem is directed towards Objective 10 (the residue theorem).

Problem 7. Using the residue theorem, compute

$$\int_{-\pi}^{\pi} \frac{1}{2+\sin\theta}\, d\theta.$$

The above problem is directed towards Objective 10 (the residue theorem).