Practice midterm 2

Complex analysis, lecture 4

October 8, 2025

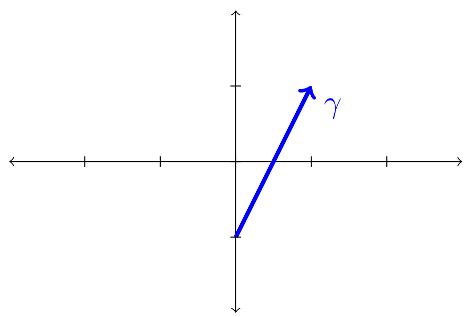
As usual, be sure to include your method, and remember to write your name.

Problem 1. Let γ be the circle of radius 3 centered at the origin. Compute

$$\int_\gamma \frac{z-1}{z^2}\,dz.$$

The above problem is directed towards Objective 5 (complex integration).

Problem 2. Consider the path γ given by the straight line segment from $-i$ to $i+1$.

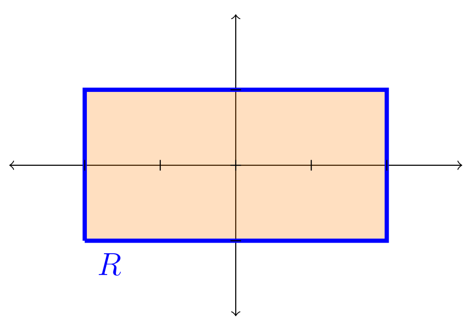


Compute

$$\int_\gamma z e^{z^2}\,dz.$$

The above problem is directed towards Objective 5 (complex integration).

Problem 3. Let R be the rectangle with corners at $-2-i$, $-2+i$, $2-i$, $2+i$:



Compute

$$\int_{\partial R} \frac{z^2+1}{z^4-z^3}\,dz.$$

Hint/timesaver: you may find the formulas $g'(z) = 1 - \frac{2}{(z-1)^2}$ and $g''(z) = \frac{4}{(z-1)^3}$ useful, where $g(z) = \frac{z^2+1}{z-1}$.

The above problem is directed towards Objective 6 (Cauchy's integral theorem and formula).

Problem 4. Let $f : D \to \mathbb{C}$ be a smooth function. For $z_0 \in D$, let $g(z) = f(z) - f(z_0)$. Show that

$$\int_{\partial D} g(z)\, dz = \int_{\partial D} f(z)\, dz.$$

The above problem is directed towards Objective 6 (Cauchy's integral theorem and formula).

Problem 5. Suppose that $f : \mathbb{C} \to \mathbb{C}$ is an analytic function such that $|f(z)| \leq |z|^n$. Conclude that $|f^{(n)}(0)| \leq n!$.

The above problem is directed towards Objective 7 (consequences of Cauchy's theorem and formula).

Problem 6. Let D be the disk of radius 1 centered at the origin. Verify Pompeiu's formula for $f(z) = |z|^2$ at $z_0 = 0$.

The above problem is directed towards Objective 7 (consequences of Cauchy's theorem and formula).