Practice midterm 1

Complex analysis, lecture 4

September 15, 2025

As usual, be sure to include your method, and remember to write your name.

Problem 1. Find a square root of $1 - i$.

The above problem is directed towards Objective 1 (complex numbers and algebra).

Problem 2. Recall that one formulation of the fundamental theorem of algebra is the statement that if $p(z)$ is a nonconstant polynomial with complex coefficients, then the equation $p(z) = 0$ has at least one solution in $\mathbb{C}$. Show by example that the analogous statement for the real numbers is not true, i.e. find a nonconstant polynomial $p(x)$ with real coefficients such that $p(x) = 0$ has no real solutions. Show that for this example, $p(z) = 0$ does have a complex solution.

The above problem is directed towards Objective 1 (complex numbers and algebra).

Problem 3. Given a complex number $z \neq 0$, choose a way to write it unambiguously in polar form. Using this, write down both branches of the complex square root of z.

The above problem is directed towards Objective 2 (branch cuts and Riemann surfaces).

Problem 4. Where are the branch points of $\frac{\sqrt{z^2-4}}{z}$? What are the phase factors at each?

The above problem is directed towards Objective 2 (branch cuts and Riemann surfaces).

Problem 5. Show that if $z \mapsto f(z)$ and $z \mapsto \overline{f(z)}$ are both analytic, then f is constant.

The above problem is directed towards Objective 3 (analytic functions and the Cauchy–Riemann equations).

Problem 6. Verify using the Cauchy–Riemann equations that $f(z) = z^2$ is analytic on $\mathbb{C}$.

The above problem is directed towards Objective 3 (analytic functions and the Cauchy–Riemann equations).

Problem 7. If $f : \mathbb{C} \to \mathbb{C}$ is an analytic function, is it possible for its real part to be $\mathrm{Re}(f(z)) = |z|^2$?

The above problem is directed towards Objective 4 (harmonic functions and conformal mappings).

Problem 8. Let $f : \mathbb{C} \setminus \{0\} \to \mathbb{C}$ be given by $f(z) = \frac{1}{z}$. At what points is this conformal? Is it a conformal mapping?

The above problem is directed towards Objective 4 (harmonic functions and conformal mappings).