Homework 5

Complex analysis, lecture 4

Due October 13, 2025 by 11:59 PM

As usual, you may use any resources to solve these problems except where stated otherwise, with the exception of computational software/generative AI and posting these problems anywhere to be answered by others. Collaboration is encouraged, but everyone should write their own solutions. Write the names of any collaborators or sources used at the top of your homework. If you did not use any sources, write "sources used: none."

If you find any errors in either the homework or the lecture notes, please let me know, even if you are unsure whether it is an error or not.

As on most math problems, the mathematics is the issue, not the answer: whether you have a correct method is more important then whether you get to the correct number at the end, so include your method!

You do not have to simplify your answers completely (so for example $\frac{2}{2}$ is fine), but you do need to do all the computations (so for example if the problem is "find the largest value of $f(x)$," the answer "$f(3)$" is incomplete; you would also need to evaluate f at 3).

Problem 1. Say that a function $f : \mathbb{C} \to \mathbb{C}$ is "of polynomial growth" if there exist constants $R, C \in \mathbb{R}_{>0}$ and $d \in \mathbb{Z}_{\geq 0}$ such that for all $|z| \geq R$, $|f(z)| \leq C|z|^d$. Using the Cauchy estimates, show that if f is of polynomial growth and analytic, then so is $f^{(n)}$ for every $n \geq 0$.

The above problem is directed towards Objective 7 (consequences of Cauchy's theorem and formula).

Problem 2. Give an example of a domain D, a continuous function $f : D \to \mathbb{C}$, and a rectangle $R \subset D$ with sides parallel to the coordinate axes such that you can show (by explicit computation) that

$$\int_{\partial R} f(z)\, dz \neq 0,$$

so that f fails the condition of Morera's theorem.

The above problem is directed towards Objective 7 (consequences of Cauchy's theorem and formula).

Problem 3. Let $f(z) = |z|^2$ and $D = \{z : |z| < 2\}$.

(a) Compute $\frac{\partial f}{\partial \bar{z}}$.

(b) Use Pompeiu's formula to write

$$\iint_D \frac{z}{z-1}\, dx\, dy$$

in terms of an integral over ∂D and a value of f.

(c) Compute the integral and value from part (b) to obtain an evaluation of the integral $\iint_D \frac{z}{z-1}\, dx\, dy$.

The above problem is directed towards Objective 7 (consequences of Cauchy's theorem and formula).

Survey. Estimate the amount of time you spent on each problem to the nearest half hour.

	Time Spent
Problem 1	
Problem 2	
Problem 3	

Please feel free to record any additional comments you have on the problem sets and the lectures, in particular, ways in which they might be improved.