Homework 1

Complex analysis, lecture 4

Due September 8, 2025 by 11:59 PM

As usual, you may use any resources to solve these problems except where stated otherwise, with the exception of computational software/generative AI and posting these problems anywhere to be answered by others. Collaboration is encouraged, but everyone should write their own solutions. Write the names of any collaborators or sources used at the top of your homework. If you did not use any sources, write "sources used: none."

If you find any errors in either the homework or the lecture notes, please let me know, even if you are unsure whether it is an error or not.

As on most math problems, the mathematics is the issue, not the answer: whether you have a correct method is more important then whether you get to the correct number at the end, so include your method!

You do not have to simplify your answers completely (so for example $\frac{2}{2}$ is fine), but you do need to do all the computations (so for example if the problem is "find the largest value of $f(x)$," the answer "$f(3)$" is incomplete; you would also need to evaluate f at 3).

All problems in this homework are directed towards Objective 1 (complex numbers and algebra).

Problem 1. Verify that if z, z' are complex numbers, then:

(a) $|z| = |\overline{z}|$;

(b) $\overline{z + z'} = \overline{z} + \overline{z'}$;

(c) $\overline{z \cdot z'} = \overline{z} \cdot \overline{z'}$.

Problem 2. Using the identity

$$e^{i(a+b)} = e^{ia} \cdot e^{ib}$$

and the definition

$$e^{i\theta} = \cos(\theta) + i\sin(\theta),$$

prove the formulas

$$\cos(a + b) = \cos(a)\cos(b) - \sin(a)\sin(b), \qquad \sin(a + b) = \cos(a)\sin(b) + \sin(a)\cos(b).$$

Problem 3. Express the following complex numbers in both polar and Cartesian form, i.e. as $z = re^{i\theta}$ and as $z = x + iy$ for real numbers r, θ, x, y.

(a) $\frac{i}{1+i}$

(b) $\left(\frac{1+i}{\sqrt{2}}\right)^{25}$

(c) $1 + e^{2\pi i/3}$

Problem 4. If the point $P = (X, Y, Z)$ on the sphere corresponds to z under the stereographic projection, show that the antipodal point $-P = (-X, -Y, -Z)$ corresponds to $-1/\overline{z}$.

Survey. Estimate the amount of time you spent on each problem to the nearest half hour.

	Time Spent
Problem 1	
Problem 2	
Problem 3	
Problem 4	

Please feel free to record any additional comments you have on the problem sets and the lectures, in particular, ways in which they might be improved.