Practice problems for midterm 2

Calculus II, section 3

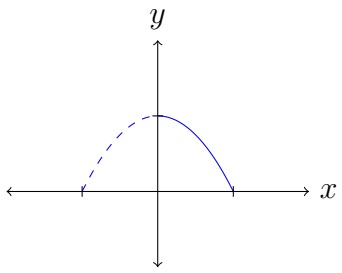
March 26

These are practice problems for the content of the second midterm. This is *not* a practice test, and you should not expect it to necessarily approximate the test in either length or difficulty; the problems on the test will likely be shorter and easier, at least on average. However, if you know the material well enough to be able to solve these problems, you are well-prepared for the midterm.

We'll go over these problems, among others, in class on Monday; full written solutions will be posted Monday night for your use in studying. I encourage you to attempt them prior to that on your own.

Problem 1. Consider the parametric curve $x = t^3$, $y = 3t^2$. Compute the arc length of the segment of the curve between $t = 0$ and $t = 1$.

Problem 2. Consider surface given by rotating the curve $y = 1 - x^2$ between its y-intercept and x-intercept around the y-axis.



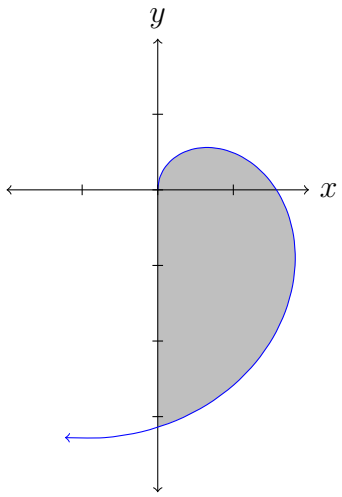
Find the surface area of this surface.

Problem 3. Consider the complex-valued function $f(t) = t \sin t + it \cos t$ for real numbers t.

(a) Write $f(t)$ in polar coordinates, i.e. write $f(t) = r(t)e^{i\theta(t)}$ for some functions $r(t)$ and $\theta(t)$.

(b) Your answer to part (a) should let you describe the same curve in the real plane by the polar curve $r = \frac{\pi}{2} - \theta$ with $\theta = \frac{\pi}{2} - t$, graphed below. Find the area bounded by

this curve between $t = 0$ and $t = \pi$, so θ between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.



Problem 4. Find a family of solutions to the differential equation $3y^2y' = 2x - 1$.

Problem 5. Find solutions to the following differential equations, where x is a function of a variable t:

(a) $x'' + x = t$ with $x(0) = x(\frac{\pi}{2}) = 0$;

(b) $tx' + x = \sin t$, with $x(1) = 0$.

Problem 6. Use the Laplace transform to solve $x'' - x = 1$, with $x(0) = x'(0) = 0$.