Homework 4

Calculus II, section 3

Hard due date: 11:59 PM Sunday February 13, 2022

+2 extra credit points if turned in by 11:59 PM Saturday February 12

+5 extra credit points if turned in by 11:59 PM Friday February 11

As usual, you may use any resources to solve these problems except where stated otherwise, with the exception of computational software and posting these problems anywhere to be answered by others. Collaboration is encouraged, but everyone should write their own solutions. Write the names of any collaborators or sources used at the top of your homework. If you did not use any sources, write "sources used: none."

Any error in either the lecture notes or the homework is worth up to 5 points of extra credit to the first person to spot it, depending on the severity of the error; email me (cbz2106@columbia.edu) if you find one. (You do *not* lose points for incorrectly pointing out an error, so please do not hesitate!)

As on most math problems, the mathematics is the issue, not the answer: whether you have a correct method is more important then whether you get to the correct number at the end, so include your method!

Problem 1. For each of the following improper integrals, determine whether it converges and if so compute its value. (25 pts each)

(a) $\int_0^{\pi/4} \frac{\sqrt{\cot(x)}}{\cos^2(x)}\,dx$, where $\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$

(b) $\int_1^{\infty} \frac{x+1}{2x^2-3x+2}\,dx$

Problem 2. Determine whether the following improper integrals converge. You do not need to find their value if they do. (20 pts each)

(a) $\int_1^{\infty} e^{-x}\log(x)\,dx$

(b) $\int_0^2 \frac{1}{\sqrt{x(2-x)}\log x}\,dx$

Problem 3. Give an example of a function $f(x)$ such that $\int_{-\infty}^{\infty} f(x)\,dx$ converges. (10 pts)

Survey (optional). Complete the following survey by rating each problem you attempted on a scale of 1 to 10 according to how interesting you found it (1 = "mind-numbing," 10 = "mind-blowing"), and how difficult you found it (1 = "trivial," 10 = "brutal"). Also estimate the amount of time you spent on each problem to the nearest half hour.

	Interest	Difficulty	Time Spent
Problem 1			
Problem 2			
Problem 3			

Please feel free to record any additional comments you have on the problem sets and the lectures, in particular, ways in which they might be improved.