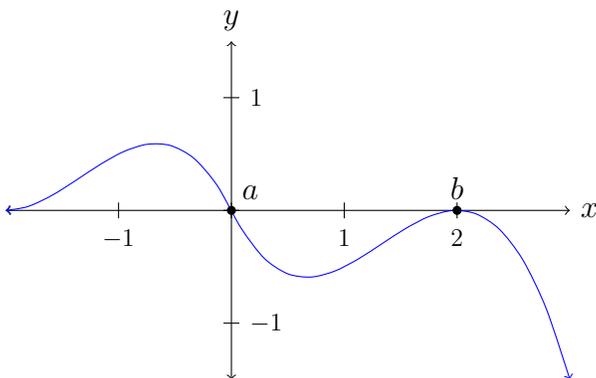# Pre-class worksheet 8: Rolle's theorem

Calculus I, section 10
Due November 2, 2023 by 4:10 PM

Suppose that $f(x)$ is differentiable between two real numbers a and b, and $f(a) = f(b) = 0$.

One thing we can see visually is that since the function returns to the same value at a and b, it must achieve a maximum or a minimum in between them: there must be some extreme point that it reaches before turning around and going back. Either way, there must be some point c between a and b where we have $f'(c) = 0$. This is Rolle's theorem: if $f(a) = f(b) = 0$ and f is differentiable between a and b, then there is some c between a and b such that $f'(c) = 0$. And already we've basically proven it: since f returns to the same value, it must achieve a maximum or a minimum somewhere, so by our knowledge of optimization the derivative must vanish somewhere.

Problem 1. For example, take $f(x) = 2x - x^2$, which has $f(0) = f(2) = 0$. Rolle's theorem predicts that there should be some point c between 0 and 2 with $f'(c) = 0$; find it.

Problem 2. Using Rolle's theorem, show that if $f(x)$ is a differentiable function with critical points at a and b (i.e. $f'(a) = f'(b) = 0$) then there must be an inflection point c (i.e. $f''(c) = 0$) between a and b.