Pre-class worksheet 6: differentiability

Calculus I, section 10
Due October 5, 2023 by 4:10 PM

In class, we saw an example of a function whose derivative did not exist at a certain point, namely $\sin(\frac{1}{x})$ at $x = 0$. This is an example of the phenomenon of *non-differentiability*. The derivative $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ is defined by a limit, and as we have seen limits may or may not exist; so the derivative may or may not exist. If $f'(x)$ exists as a limit, we say that f is differentiable at x; if not, it is not differentiable at x. Just like for continuity, we often say that a function $f(x)$ is differentiable to mean it is differentiable everywhere on its domain.

In order for a function to be differentiable at a point x, it must exist and be continuous at x (or have a removable discontinuity, since then the limit still exists). However, it is possible for a function to both exist and be continuous at a point, yet not be differentiable.

Problem 1. Consider the function $f(x) = |x|$. By computing both one-sided limits, show that $f(x)$ is not differentiable at $x = 0$.

Problem 2. On the other hand, show that $f(x) = |x|^3$ is differentiable at $x = 0$.

Challenge problem (1 point). Give a formula for $\frac{d}{dx}|x|^3$.