Pre-class worksheet 4: derivatives of rational functions

Calculus I, section 10

Due September 28, 2023 by 4:10 PM

In class, we talked about derivatives of polynomials, using linearity together with the power rule: $\frac{d}{dx}x^n = nx^{n-1}$. We proved this rule for positive integers n using the binomial theorem, but claimed that it actually holds for all real numbers n.

Problem 1. Show directly using the limit definition of the derivative that $\frac{d}{dx} = -\frac{1}{x^2}$, agreeing with the output of the power rule for $n = -1$.

Problem 2. Use this calculation to find the first-order approximation of $\frac{1}{1.003}$: first choose an x_0 where $f(x) = \frac{1}{x}$ is easy to evaluate such that x_0 is close to our value of $x = 1.003$, and then use linear approximation to estimate $f(1.003)$.