

# Calculus I, section 10: syllabus

Fall 2023

**Time and location:** Tuesdays and Thursdays 4:10 - 5:25 PM in 407 Mathematics

**Instructor:** Avi Zeff, email avizeff@math.columbia.edu

**Office hours:** TBD

**TA:** TBD

Welcome to calculus! In this course we'll study the language of functions, motion, and change. This language is fundamental to fields like physics, economics, engineering, and of course mathematics itself, as well as many others. You are all probably coming from very different backgrounds and with different goals for this class; the good news is that you can all succeed. (There is no bad news, because we are doing math and so everything is good.)

## Summary

There are four main areas we will cover:

- Functions and limits
- Derivatives
- Applications of derivatives
- Integrals and their relationship to derivatives

By the end of the course, students should have a good working understanding of mathematical functions; be able to compute limits and derivatives of various kinds of functions, and apply these skills to concrete problems such as optimization; and understand the relationship between integration and differentiation, and be able to compute simple integrals and apply integrals to problems such as finding areas or average values.

A more detailed course outline can be found at the end of this document.

## Prerequisites

The prerequisite for this class is precalculus or equivalent: in other words, you should be comfortable manipulating algebraic expressions (for example to solve equations) and be familiar with the general notions of functions and graphs, as well as examples of functions such as polynomials, exponential and logarithmic functions, and trigonometric functions such as sine and cosine, although we will take a little time at the beginning of the course to review these notions. If you are not sure whether you have the prerequisites, email me or come to my office hours.

## Textbook

The nominal textbook for this course is *Calculus: Early Transcendentals*, by James Stewart. If you feel that access to a textbook will help you learn, you are free to purchase it; it can also be found online or in the library. If you would like access to the textbook and are having trouble getting it, let me know.

That said, use of the textbook in any form is not required: although problems may be drawn from it, their statements will always be included in the assignment, and no readings from the book will be assigned. Lecture notes will be posted before each class; you will not be responsible for any material not included in these. (Pre-class readings from lecture notes or other materials made available on the class website may be assigned.)

For those who find textbooks useful, in addition to the lecture notes there exist many good calculus textbooks, including Strang's *Calculus*, which the author has made freely available online.

We will not use WebAssign. (You also do not need a calculator for this course, and one will not be permitted on exams.)

## Workflow

A pre-class worksheet will be assigned before most classes, often accompanying a brief reading. These are intended to prepare you for the class, and are graded only for good-faith completion. They are the only type of assignment in this class that cannot be reattempted (or submitted late): if you miss one, there is nothing to be gained by completing it later. (However if there are extenuating circumstances making you unable to complete some worksheets, on a case-by-case basis the effect on your grade can be mitigated.) Worksheets are due by class time (4:10 PM) every class for which they are assigned.

In class, we will do a mixture of lecture and active work, either alone or in groups, and then discuss the problems together. Every few lectures, there will be a short in-class quiz (advertised in advance). Due to the reattempt policy (see below), these should be fairly low-stakes.

After class, you will apply the concepts we learn on longer-form homework, graded for correctness.

## Course structure

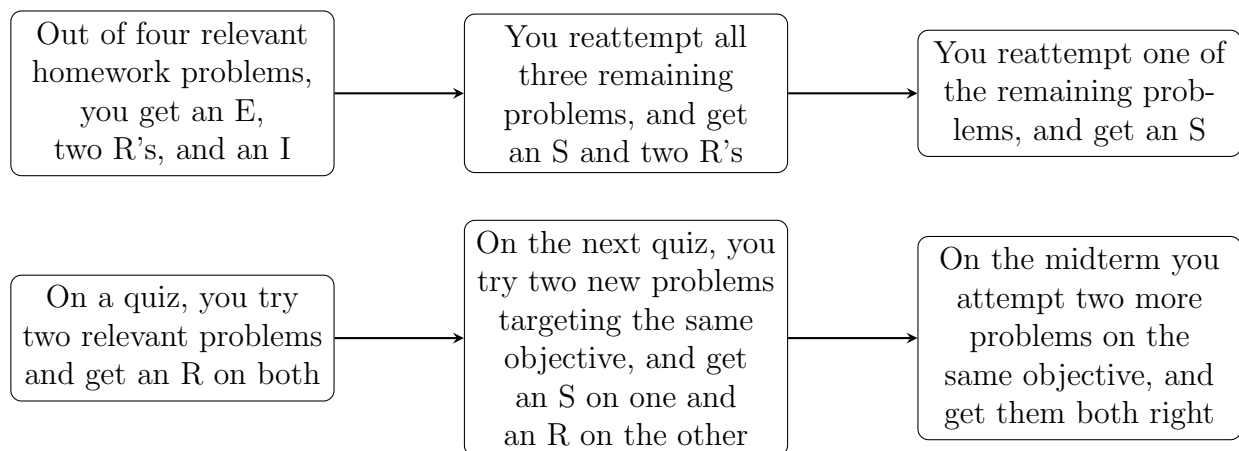
This course will be taught via “standards-based learning,” the central idea of which is that there is a set of objectives which you are here to learn, and the class should be taught in such a way as to optimize the amount of these (both in number and degree) that you learn by the end of the semester, and graded based on how many you have learned to a satisfactory standard. Concretely, this means that both the class structure and grading may be different from what you are used to:

- Your grade will be determined by the number of objectives for which you have demonstrated achievement. Higher grades require a certain amount of “exemplary work” as well as sufficient completion of pre-class work (see below for a detailed table).
- In order to demonstrate achievement of each objective, you need to successfully solve problems on that topic **both** on the homework **and** on the exams/quizzes.
- However, if you don't manage to solve a problem correctly on the first attempt, **you can re-attempt it without penalty** (for homework problems) or replace it with a problem on the same objective on a later exam/quiz.

Each problem will be graded either E (exemplary), S (satisfactory), R (revisions needed), or N (not assessable), with comments to help you revise as needed. In order to demonstrate achievement of an objective, you need at least an S on the relevant homework problems (dropping up to one homework problem per objective) and at least two quiz or exam problems.

Examples of E-level, S-level, and R-level work are posted on the website. Work which is significantly incomplete, does not address the question, or is illegible to the grader will receive a mark of N, and may receive fewer or no comments.

Below is a flowchart showing one possible path to achieving an objective.



At the end of both diagrams, your homework marks are E, S, S, and R, so at least three of them are at S or above, which is all but one of the relevant homework problems; and you've gotten three S marks on quizzes or exams, which is more than the two required. Therefore you have achieved this objective after finishing **both** flowcharts. (You have additionally earned one challenge point, see below.)

To keep the workload manageable for all of us, **you are limited to re-attempting at most three homework problems per week**, where weeks begin at 12:00 AM Sunday and end at 11:59 PM Saturday. (You cannot save up re-attempts for later in the semester, so there is no reason to not re-attempt early on.) Future quizzes will continue to have problems from past objectives, so if you don't achieve a mark to your satisfaction you can continue to solve problems on that objective on future quizzes. Once you have achieved two S marks on quiz or exam problems for a given objective, you may skip problems on that objective on future quizzes and exams.

Marks of E (exemplary) are given for outstanding solutions to homework, quiz, or exam problems which are not only correct but also clear, complete, easy to read and follow, use correct terminology and notation, and in short could be used as examples to teach from. These are given entirely at the discretion of the grader. Marks of E can only be achieved on initial submissions of homework problems, not on revisions, though they can be achieved on quiz or exam problems regardless of any previous attempts on the same objective; however you can earn at most two challenge points for exemplary quiz or exam solutions on any one objective. Although they are not necessary in order to get credit for achieving an objective, they count towards "challenge points," which are necessary to achieve the highest

grades in the class. Challenge points can also be achieved through challenge problems, optional problems appearing on homework assignments, quizzes, and exams, which may be worth multiple points depending on their difficulty. Challenge problems can be reattempted; however you can only earn challenge points on a given problem if the regular problems on that objective (on both the current assignment and any previous ones) are correct, so make sure to prioritize the required work before working on challenge problems!

## Homework

Homework will be weekly, generally due on Tuesday in (or by, if submitted digitally) class (i.e. at 4:10 PM). You should expect to spend a while on these, in the neighborhood of 2-5 hours; much of learning is done through exercises. Collaboration is strongly encouraged, but everyone should write their own solutions; **write on your homeworks anyone you have worked with**. (Note, though, that contributions from different collaborators should be roughly equal; if you find that you are typically doing more or less than your collaborators, consider finding a different group.)

Unless otherwise specified, you may use any and all resources (again, **citing any sources you have used**), including any textbooks you have access to, your classmates and friends, the help room, office hours (you should come to these!), or the internet, with the following exceptions:

- do not post the problem on any website to be answered by someone else;
- do not use computer algebra systems (e.g. WolframAlpha, SageMath, Mathematica, integral-calculator.com, generative AI, etc.) to do your computations unless otherwise specified. That said, these are all useful resources I encourage you to familiarize yourself with for any purpose other than homework for this class (and naturally any other classes with similar policies).

The reason for these exceptions is that while using other resources does not prevent you from learning from doing computations, outsourcing those computations entirely does.

For any homework problem on which you are dissatisfied with your grade, you should carefully review the feedback and reattempt the problem, and resubmit it via the resubmission assignment on CourseWorks. Resubmissions made in the incorrect place (e.g. the original assignment) may not be graded, or graded late. You can submit reattempts once per week for each problem, up to a maximum of three problems per week; if your reattempt is not successful, you can continue to reattempt the same problem as many times as needed through the end of the semester. The last day to submit reattempts is the date of the final exam. Keep in mind that as problems pile up, the three-problem limit will become more and more of an issue towards the end of the semester, so it is best to stay on top of revisions early on.

We will attempt to have all homework graded within one week of its submission, including reattempts. Late homework may be graded late.

## Exams and quizzes

There will be one midterm and a final exam, as well as regular quizzes throughout the semester. Quizzes will typically be about 15-20 minutes, and will always be announced in

advance (tentative dates are on the schedule below, and any changes will be announced); the midterm will be a full class session, i.e. 75 minutes; and the final exam will be three hours. Functionally, exams are just longer quizzes: they are opportunities to demonstrate achievement of objectives, and both will include problems on all objectives covered thus far. You will have much more time for the exams, so they are an opportunity to catch up if you are missing a number of problems, but the problems themselves will be very similar in style and difficulty to quiz problems. Both quizzes and exams will also have challenge problems of varying difficulties.

For any exam problem on which you are dissatisfied with your grade, you should carefully review the feedback and, when ready, attempt problems on the same objective on future quizzes or exams.

There will be no homework the weeks of the midterm (or, of course, the final exam). This unavoidably means that some topics assessed on the midterm will not have been assessed on homework at that time (though they will be after the midterm), so be especially careful to study those.

## Grading

Your grade will be determined by the number of objectives you achieve, together with challenge points. There are 14 objectives, listed below. All of them are important (or else we would not spend time on them!), but some are crucial that you know in order to be able to progress to future classes; these have been marked as (CORE) below. In order to pass the class (i.e. with a C-) you must achieve all six of these core objectives, up to some flexibility as detailed below. To achieve a B or A-level grade, you must demonstrate a certain amount of outstanding work, through exemplary solutions or challenge problems.

Thus the grade requirements are as follows:

| Grade | Objectives achieved   | Challenge points |
|-------|---|------------------|
| A+    | 14, including all core objectives   | $\geq 50$        |
| A     | $\geq 13$ , including all core objectives                                     | $\geq 40$        |
| A-    | $\geq 12$ , including all core objectives                                     | $\geq 30$        |
| B+    | $\geq 11$ , including all core objectives                                     | $\geq 20$        |
| B     | $\geq 10$ , including all core objectives                                     | $\geq 10$        |
| B-    | $\geq 9$ , including all core objectives                                      | $\geq 5$         |
| C+    | $\geq 8$ , including all core objectives                                      | $\geq 0$         |
| C     | $\geq 7$ , including all core objectives                                      | $\geq 0$         |
| C-    | All 6 core objectives,<br>OR<br>$\geq 8$ , including $\geq 4$ core objectives | $\geq 0$         |
| D     | $\geq 6$ , including $\geq 3$ core objectives                                 | $\geq 0$         |

Your grade will be the highest for which you've fulfilled both requirements. A grade of F will be assigned if the requirements for a D are not met. In addition, for every four pre-class worksheets you miss, your overall grade will be lowered by one-third of a letter grade, i.e. B to B-, B- to C+, etc. For example, if you've achieved 13 objectives and 32 challenge

points, your grade would be an A-, as that is the highest grade for which you've fulfilled the requirements. If on top of that you were missing four worksheets, your grade would be lowered to B+.

In order to achieve an objective, as described above you need at least an S on every homework problem targeting that objective except for one (so for example if there are four relevant problems, you need at least an S on at least three of them) as well as at least an S on at least two quiz or exam problems targeting that objective.

I reserve the right to modify this grading scheme over the course of the semester, but only ever in your favor (e.g. I could change the requirements for a B to only requiring 7 challenge points instead of 10, but will not change it to requiring 15).

The objectives are as follows.

- (1) You understand and can manipulate and use algebraic expressions and functions. (CORE)
- (2) You can calculate and use limits of various kinds of functions, including infinite limits, and explain their meaning. (CORE)
- (3) You can determine whether a function is continuous at each point and describe its discontinuities and asymptotes.
- (4) You understand the meaning of the derivative and the relationship between the limit definition and the geometric picture. (CORE)
- (5) You can directly compute examples via the limit definition and determine whether a function is differentiable at each point.
- (6) You understand and can apply linearity, the product, quotient, and chain rules, and implicit differentiation to calculate derivatives. (CORE)
- (7) You can combine multiple rules in order to compute derivatives of complicated functions, including combinations of polynomials, trigonometric functions, exponentials, or their inverses.
- (8) You can apply the concept of the derivative and the second derivative test to find extrema of functions.
- (9) You can compute limits using derivatives via L'Hopital's rule.
- (10) You can formulate real-world problems (such as in optimization, dynamics, or approximation) in the language of calculus, and can use derivatives (including techniques such as maximization/minimization, related rates, and Newton's method) to solve them.
- (11) You understand the meanings of the antiderivative and definite integral, and can compute simple examples from the definitions. (CORE)
- (12) You can apply  $u$ -substitution in combination with differentiation to compute certain kinds of antiderivatives.

- (13) You understand the relationship between integration and differentiation through the fundamental theorem of calculus. (CORE)
- (14) You can apply integration to real-world problems such as computing areas and average values.

## Course policies

### Attendance

Attendance is not mandatory, in the sense that I will not be taking attendance and it is not part of your grade, but it is expected as a part of the class. Empirically, students tend to do better when they come to class. However if you need to miss a class for any reason, **you do not need to inform me.**

If you miss a quiz, there will be opportunities to do problems on the same objectives on future quizzes and exams; if you feel a pressing need to take the quiz before that and cannot make it to class the day of the quiz, contact me **beforehand** (or as soon as possible afterwards in case of illness) and we can work something out. Frequently missing quizzes is likely to make it more difficult for you to achieve objectives, so if I notice that you're often missing them I may reach out to you.

### Deadlines and extensions

If you are unable to turn in your homework by the official deadline, please contact me **at least 24 hours in advance** with a request for an extension; you must specify when you would like the new deadline to be. I expect to grant all such requests, within reason, but try not to do this more than you have to in order to make the life of our grader (who may be me) easier.

If your request for an extension is within 24 hours of the deadline or does not specify a new deadline, it probably will not be granted except in extenuating circumstances; however you'll still have the opportunity to resubmit your homework as a revision. This will however count toward the three-problem limit per week, so you may need to submit the problems separately over several weeks (which would prevent you from doing other revisions during that period). As such it's best for everyone if you turn in your homework on time.

### Technology requirements

You do not need any particular technological devices for this class other than a writing implement (for tests and quizzes, at least); in particular you do not need a calculator. However, many materials will by default only be made available online, via CourseWorks, and homework submissions will also be online. If this presents a difficulty for you contact me and I'll make the materials available on paper as needed.

## COVID-19 policies

Classes will be in person, but please do not come to class if you are feeling sick or test positive: lecture notes will be available, class can be streamed or recorded as needed, and I

will be happy to help you make up the material. If you are sick and unable to do the work for a prolonged period, contact me to work out a way to make up the work: it is important to do the work for each section of the class, since that is the way to learn the material and the later portions of the class will build on the earlier ones, but when necessary we can figure out how to reduce the workload to be manageable without having to work through illness. Similarly, please do not attend exams if you are ill or have recently tested positive for COVID-19: we will figure out solutions if these situations arise.

Classes may or may not be videotaped and recorded and/or streamed; we'll discuss what we want to do in class. The first few classes will be recorded for anyone joining the class late.

## **Academic Honesty Policy**

Please read (and follow) the Columbia University Undergraduate Guide to Academic Integrity. If you are ever unsure whether something is allowed, please ask me first—you will never be penalized for asking.

## **Accessibility and accommodations**

Please let me know if there is anything I can do to make this course more accessible to you, or if aspects of the course are excluding you, and we can work together to develop strategies to improve the class. If you think you may need official accommodations, such as extended time on exams, I encourage you to contact the Office of Disability Services for an accommodation letter.

## **Feedback**

There will be some formal opportunities to give feedback, such as on homeworks. In addition, feel free to give me comments on the class at any time, via email or in person; if you are not comfortable doing either, you can give me anonymous feedback via this form.



## Tentative course outline

As promised:

|              |  |
|--------------|--|
| September 05 | Class overview, review of algebra and functions            |
| September 07 | More functions: examples, inverses, iteration              |
| September 12 | Introduction to limits                                     |
| September 14 | Limit laws and continuity; quiz                            |
| September 19 | Computing limits and the squeeze theorem                   |
| September 21 | Asymptotes and discontinuities                             |
| September 26 | Introduction to derivatives; quiz                          |
| September 28 | Product and quotient rules                                 |
| October 03   | Chain rule   |
| October 05   | Implicit differentiation and inverse functions; quiz       |
| October 10   | Exponentials and logarithms                                |
| October 12   | Differentiating general functions                          |
| October 17   | Review   |
| October 19   | Midterm  |
| October 24   | Related rates and extrema                                  |
| October 26   | Maximization and the second derivative test                |
| October 31   | Optimization   |
| November 02  | The mean value theorem and L'Hopital's rule; quiz          |
| November 07  | <i>Election Day</i>  |
| November 09  | Graphs and Newton's method                                 |
| November 14  | Antiderivatives  |
| November 16  | Examples and $u$ -substitution; quiz                       |
| November 21  | Definite integrals and the fundamental theorem of calculus |
| November 23  | <i>Thanksgiving</i>  |
| November 28  | Applications of integration                                |
| November 30  | Assorted related topics; quiz                              |
| December 05  | Flex day   |
| December 07  | Review   |
| TBD          | Final exam   |

“Assorted related topics” can be any additional calculus material which we would not otherwise cover in this class, depending on class interest (e.g. methods of integration, differential equations, Taylor series...). The “flex day” gives us room for any overflow—if some topics take longer than expected or we’re forced to miss a class, we can adapt without having to rush too much; otherwise it can be additional bonus content (or a day off).