Calculus I, section 10: examples of standards

Exemplary

Exemplary, or E, work is not only completely correct, without even small mistakes, but also clear, complete, easy to read and follow, use correct terminology and notation, and in short could be used as examples to teach from with no danger of misleading or confusing its audience. These are given entirely at the discretion of the grader.

For example, suppose the problem is the following (say we are learning how to solve quadratic equations):

Problem. Find all *positive* solutions to $2x^2 + x = 3$.

An example of an exemplary solution is as follows:

Solution. Moving the 3 to the other side of the equation, this is $2x^2 + x - 3 = 0$. Now we can factor the left-hand side:

$$2x^2 + x - 3 = (x - 1)(2x + 3) = 0.$$

Since the product of two numbers can only be zero if at least one of them is zero, we have either $x - 1 = 0$ or $2x + 3 = 0$. Solving these equations gives either $x = 1$ or $x = -\frac{3}{2}$. Since the problem asks for positive solutions, only $x = 1$ works, and so the answer is just $x = 1$.

This answer is completely correct, explains each step concisely but clearly, and is legible and easy to follow; if a student was unsure how to solve this equation, we could use this solution as a guide to walk them through it.

Satisfactory

Satisfactory, or S, work completely and correctly solves the given problem, and is clearly and legibly explained. There may be at most one or two very minor errors such as typos or misspellings, but nothing that significantly affects the mathematical content.

For example, suppose the problem is the following (say we are learning how to solve quadratic equations):

Problem. Find all *positive* solutions to $2x^2 + x = 3$.

An example of a satisfactory solution is as follows:

Solution. Moving the 3 to the other side of the equation, this is $2x^2 + x - 3 = 0$. Now we can factor the left-hand side: $(x - 1)(2x + 3) = 0$, so we have either $x - 1 = 0$ or $2x + 3 = 0$, i.e. (adding 1 to both sides) $x = 1$ or (subtracting 3 and dividing by 2) $x = -\frac{2}{3}$. Since the problem asks for positive solutions, only $x = 1$ works, and so the answer is just $x = 1$.

This answer is almost completely correct, and it is (reasonably) easy to follow what is being done. However it contains a small error: the solution to $2x+3=0$ is actually $-\frac{3}{2}$, not $-\frac{2}{3}$. Since both are negative, this doesn't affect the final result; more importantly, it is an arithmetic mistake, not an algebra one, and doesn't change the fact that the student clearly understands conceptually how to solve the problem.

Another example of a satisfactory solution is as follows.

Solution.

$$2x^2+x=3 \qquad 2x^2+x-3=0$$
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{1^2-4\cdot 2\cdot(-3)}}{2\cdot 2}=\frac{-1\pm\sqrt{25}}{4}$$

which is $-\frac{3}{2}$ or 1. Since 1 is positive it is the only correct answer.

This solution gets the right answer, and the explanation is reasonably clear (leaving out the words is fine, though sometimes risky). However there is less explanation than in the previous example, and if there were a significant error this would make it harder to find; this is not clear enough for a student just learning this material to follow, and so falls short of exemplary. In addition, the conclusion that 1 is the only correct answer because it is positive is not, strictly speaking, correct; it is also necessary to check that $-\frac{3}{2}$ is negative. However, it's clear that this student understands what is going on and how to solve the problem, and no error is really significant.

Revisions needed

Work is graded **Revisions needed**, or R, if it is has significant errors or displays an understanding of the underlying concepts which is less than completely correct. The extent of the revisions needed will depend on the severity of the errors or misunderstandings.

Let's use the same example problem as above. Here is an example of a solution which would receive the grade R.

Solution. This is the same thing as $2x^2+x-3=0$, which we can factor as $(x-1)(2x+3)$, so either $x=-1$ or $x=\frac{2}{3}$. Only the second one is positive, so the answer is just $x=\frac{2}{3}$.

This student has correctly set up the equation and factored, but they do not completely understand the factoring method: they know that the solution is somehow related to the factors, but not why and therefore have made a sign error on each term. This will take some studying and discussion to fully understand this concept, but they do have an understanding of the concept, if incomplete, and it is possible that the student has just made a straightforward sign error. The revision for this solution should include an explanation of how to get the correct solution from the factoring and why it works.

An example requiring more extensive revisions is as follows.

Solution. The left-hand side is $x(2x+1)$, so either $x=3$ or $2x+1=3$, i.e. $x=3$ or $x=1$. Both are positive, so the solution is both.

This exhibits a major misunderstanding of factoring, and the student could have easily checked that the resulting answer is not correct. A reattempt would need to do the problem from scratch, explaining why the solution works.

Not assessable

Work is graded **not assessable** if it is significantly incomplete, does not address the question, or is illegible to the grader. An example is as follows:

Solution. $2x^2 + x = 3 = \sqrt{3 - x}$ none

It is very difficult to guess at the student's train of thought here. If the grader is feeling especially kind they might do so and try to figure out what their error probably was, marking this as R and trying to give helpful feedback. However, it is not reasonable to expect this at this level of work, and so answers like this should be expected to in general get a mark of N and probably no helpful feedback.

Solution. $x = 1$.

Although strictly speaking the final answer is correct, there is nothing here to grade: any solution necessarily includes the method, and this has none.