# Practice problems for midterm - solutions

Calculus I, section 10

October 17

These are practice problems for the content of the midterm (Objectives 1 - 7). This is *not* a practice test, and you should not expect it to necessarily approximate the test in either length or difficulty. However, if you know the material well enough to be able to solve these problems, you are well-prepared for the midterm.

Full written solutions will be posted by Tuesday for your use in studying. I encourage you to attempt them prior to that on your own.

**Problem 1.** If $f(x) = \log_4(x)$, find $\lim_{x\to 0^+} 4^{f(x)}$, if it exists.
*The above problem is directed towards Objectives 1 and 2 (functions and limits).*

**Problem 2.** Let $f(x) = \frac{\cos(x)}{x}$. Find all horizontal, vertical, or diagonal asymptotes of $y = f(x)$.
*The above problem is primarily directed towards Objective 3 (continuity and asymptotes).*

**Problem 3.** Let
$$f(x) = \begin{cases} 2x & x > 1 \\ x^2 + 1 & x < 1 \end{cases}.$$

(a) Find all real numbers $a$ where $f(x)$ has discontinuities, if any exist.

(b) Find all real numbers $a$ where $f(f(x))$ has discontinuities, if any exist.

*The above problem is directed towards Objectives 1 and 3 (functions and continuity).*

**Problem 4.** Recall that we showed in class that for $x$ near 0, we have $\cos(x) \leq \frac{\sin(x)}{x} \leq 1$, and concluded by the squeeze theorem that since $\lim_{x\to 0} \cos(x) = \lim_{x\to 0} 1 = 1$, we must also have $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$. Use these inequalities together with the squeeze theorem to similarly show that $\lim_{x\to 0} \frac{\tan(x)}{x} = 1$.
*The above problem is primarily directed towards Objective 2 (limits).*

**Problem 5.** Evaluate the limit $\lim_{x\to 0} 3^{\frac{1}{x^2}}$, if it exists.
*The above problem is primarily directed towards Objective 2 (limits).*

**Problem 6.** Is it possible for there to be a function $f(x)$ such that $f(x) + \sin(x)$ has a horizontal asymptote? If so, give an example; if not, explain why not.
*The above problem is primarily directed towards Objective 3 (continuity and asymptotes).*

**Problem 7.** Consider the function $f(x) = \frac{\sqrt{|x|}-1}{x^2-4x+3}$. Find and classify all of its discontinuities, if any exist.
*The above problem is primarily directed towards Objective 3 (continuity and asymptotes).*

**Problem 8.** Suppose $f(x) = x^5$. We know that $f(2) = 2^5 = 32$. Using the derivative of $f(x)$ at $x = 2$, give an approximation to $f(2.01)$.
*The above problem is directed towards Objectives 4 and 6 (derivatives and rules of differentiation).*

**Problem 9.** Let $f(x) = x^{\frac{1}{x+1}}$ for $x \geq 0$. Find a formula for $f'(x)$. For which $x \geq 0$ is $f(x)$ differentiable?
*The above problem is directed towards Objectives 5 and 6 (differentiability and rules of differentiation).*

**Problem 10.** If $\sin(y) = x \cos(x)$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$, $y = 0$.
*The above problem is directed towards Objectives 6 and 7 (rules of differentiation and combining rules).*

**Problem 11.** Consider
$$f(x) = \begin{cases} 2 & x \geq 5 \\ 1 & 1 < x < 5 \\ x & x \leq 1 \end{cases}.$$

Find $f'(x)$ wherever it is differentiable, and determine where it is not differentiable.
*The above problem is directed towards Objectives 4 and 5 (derivatives and differentiability).*

**Problem 12.** Differentiate $f(x) = \frac{3-x}{x^2}$.
*The above problem is directed towards Objectives 6 and 7 (rules of differentiation and combining rules).*

**Problem 13.** Let $f(x) = x^2 \sin^{-1}(x)$. Find $f'(0)$.
*The above problem is primarily directed towards Objective 7 (combining rules of differentiation).*