Homework 9

Calculus I, section 10

Due November 14, 2023 by 4:10 PM

As usual, you may use any resources to solve these problems except where stated otherwise, with the exception of computational software and posting these problems anywhere to be answered by others. Collaboration is encouraged, but everyone should write their own solutions. Write the names of any collaborators or sources used at the top of your homework. If you did not use any sources, write "sources used: none."

If you find any errors in either the homework or the lecture notes, please let me know, even if you are unsure whether it is an error or not.

As on most math problems, the mathematics is the issue, not the answer: whether you have a correct method is more important then whether you get to the correct number at the end, so include your method!

You do not have to simplify your answers completely (so for example $\frac{2}{2}$ is fine), but you do need to do all the computations (so for example if the problem is "find the largest value of $f(x)$," the answer "$f(3)$" is incomplete; you would also need to evaluate f at 3).

Problem 1. Let $f(x) = x^2 - 2$. Use Newton's method to get a numerical approximation to the positive solution of $f(x) = 0$, namely $\sqrt{2}$, to the nearest 0.001 (i.e. your answer should round to 1.414). Use a calculator to estimate the decimal approximation for your final answer (but be careful not to drop too many digits before that, as you may introduce error otherwise).

The above problem is primarily directed towards Objective 12 (assorted applications of differentiation).

Challenge problem (2 points). Give an example of an approximation problem of the form "approximate the solution to $f(x) = 0$ near x_0," for explicit choices of $f(x)$ and x_0, such that Newton's method fails. (Do not use the example from class.)

Challenge problem (3 points). Use Newton's method together with the principles of optimization we've learned to numerically approximate the point x giving the minimum value of $f(x) = e^x - \ln(x)$ for $x > 0$, using at least two steps of Newton's method.

Challenge problem (∼ 3-5 points). In class, we gave a method of approximating integrals using piecewise constant functions: we divided the interval into sub-intervals, and on each sub-interval used the constant, or zeroth-order, approximation of our function. One can also use a piecewise linear approximation; this gives a method called the trapezoid rule. Work out and explain how this should work in general, and illustrate by using the trapezoid rule to approximate

$$\int_0^1 x^2$$

using four sub-integrals (recall that in class, we found that the piecewise constant approximations gave upper and lower bounds of $\frac{15}{32}$ and $\frac{7}{32}$ to the true value of $\frac{1}{3}$).

The number of points awarded will depend on the correctness and quality of the explanation, with a correct but minimal solution worth around 3 points and a detailed and precise one (maintaining accuracy, relevance, and brevity) worth up to 5, or potentially even more.

Survey. Estimate the amount of time you spent on each problem to the nearest half hour.

	Time Spent
Problem 1	
Challenge problem 1	
Challenge problem 2	
Challenge problem 3	