Homework 7

Calculus I, section 10

Due October 31, 2023 by 4:10 PM

As usual, you may use any resources to solve these problems except where stated otherwise, with the exception of computational software and posting these problems anywhere to be answered by others. Collaboration is encouraged, but everyone should write their own solutions. Write the names of any collaborators or sources used at the top of your homework. If you did not use any sources, write "sources used: none."

If you find any errors in either the homework or the lecture notes, please let me know, even if you are unsure whether it is an error or not.

As on most math problems, the mathematics is the issue, not the answer: whether you have a correct method is more important then whether you get to the correct number at the end, so include your method!

You do not have to simplify your answers completely (so for example $\frac{2}{2}$ is fine), but you do need to do all the computations (so for example if the problem is "find the largest value of $f(x)$," the answer "$f(3)$" is incomplete; you would also need to evaluate f at 3).

Problem 1. Use linear approximation to give estimates for $f(x+1)$ given $f(x)$ when x is large for the following functions.

(a) $f(x) = \ln(x)$

(b) $f(x) = \frac{1}{x}$

The above problem is primarily directed towards Objective 10 (assorted applications of differentiation).

Problem 2. Find the rate of change $\frac{dA}{dC}$ of the area A of a circle with respect to its circumference C. (Hint: relate both A and C to the radius r, and use related rates.)

The above problem is primarily directed towards Objective 10 (assorted applications of differentiation).

Problem 3. Find and classify the critical points of $f(x) = xe^{-x^2}$.

The above problem is directed towards Objective 8 (extrema).

Problem 4. Find the global maximum of $\tan^{-1}(3x) - \tan^{-1}(x)$.

The above problem is primarily directed towards Objective 8 (extrema).

Challenge problem (3 points). Find the value of x maximizing $f(x) = x^{1/x}$ for $x > 0$. You may use without proof the fact that $\lim_{x\to\infty} \frac{\ln x}{x} = 0$ (but remember to check both limits).

Challenge problem (2 points). Choose two functions $f(x)$ and $g(x)$ (your choice, but they can't be too simple: let's say that they shouldn't be ones we've done before, shouldn't be constant or linear, and at least one must not be a polynomial) and use the method from problem 1 to estimate $f(x+1)$ and $g(x+1)$ in terms of $f(x)$, $f'(x)$, $g(x)$, and $g'(x)$. Using a calculator, compare your estimates for $f(x+1)$ and $g(x+1)$ to the true values.

Survey. Estimate the amount of time you spent on each problem to the nearest half hour.

	Time Spent
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Challenge problem 1	
Challenge problem 2	