Homework 5

Calculus I, section 10

Due October 10, 2023 by 4:10 PM

As usual, you may use any resources to solve these problems except where stated otherwise, with the exception of computational software and posting these problems anywhere to be answered by others. Collaboration is encouraged, but everyone should write their own solutions. Write the names of any collaborators or sources used at the top of your homework. If you did not use any sources, write "sources used: none."

If you find any errors in either the homework or the lecture notes, please let me know, even if you are unsure whether it is an error or not.

As on most math problems, the mathematics is the issue, not the answer: whether you have a correct method is more important then whether you get to the correct number at the end, so include your method!

You do not have to simplify your answers completely (so for example $\frac{2}{2}$ is fine), but you do need to do all the computations (so for example if the problem is "find the largest value of $f(x)$," the answer "$f(3)$" is incomplete; you would also need to evaluate f at 3).

Problem 1. Compute the derivatives of the following functions.

(a) $f(x) = \frac{1}{2x^2 - 3x + 1}$

(b) $f(x) = x^3 \sin(x^2)$

(c) $f(x) = \sqrt{4x^2 + 1}$

(d) $f(x) = \tan^{-1}(2x - 1)$

The above problem is directed towards Objectives 6 and 7 (rules of differentiation and combining rules of differentiation).

Problem 2. Give an example of a function which is continuous at all real numbers and differentiable at all real numbers except $x = 0$ and $x = 1$.

The above problem is primarily directed towards Objective 5 (differentiability).

Problem 3. Consider the function

$$f(x) = \cos(4x + 1)^3 \cdot \sin\left(\frac{1}{x - 3}\right).$$

Find its derivative $f'(x)$, and any points at which $f(x)$ fails to be differentiable. Is it continuous at those points? (Warning: the formula for $f'(x)$ is pretty complicated!)

The above problem is directed towards Objectives 5 and 7 (differentiability and combining rules of differentiation).

Problem 4. Suppose that x and y are variables related by the equation $x^2 + y^2 = 1$. Find $\frac{dy}{dx}$ at the point $x = y = \frac{\sqrt{2}}{2}$.

The above problem is directed towards Objectives 6 and 7 (rules of differentiation and combining rules of differentiation).

Challenge problem (2 points). Let f, g, and h be differentiable functions. Using the chain rule, find a formula for

$$\frac{d}{dx} f(g(h(x))).$$

Challenge problem (3 points). Suppose $f(x)$ and $g(x)$ are differentiable functions such that $f(g(x)) = \frac{1}{x}$, $g(1) = 0$, and $f'(0) = 1$. Find $g'(1)$.

Survey. Estimate the amount of time you spent on each problem to the nearest half hour.

	Time Spent
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Challenge problem 1	
Challenge problem 2	