Homework 4

Calculus I, section 10

Due October 3, 2023 by 4:10 PM

As usual, you may use any resources to solve these problems except where stated otherwise, with the exception of computational software and posting these problems anywhere to be answered by others. Collaboration is encouraged, but everyone should write their own solutions. Write the names of any collaborators or sources used at the top of your homework. If you did not use any sources, write "sources used: none."

If you find any errors in either the homework or the lecture notes, please let me know, even if you are unsure whether it is an error or not.

As on most math problems, the mathematics is the issue, not the answer: whether you have a correct method is more important then whether you get to the correct number at the end, so include your method!

You do not have to simplify your answers completely (so for example $\frac{2}{2}$ is fine), but you do need to do all the computations (so for example if the problem is "find the largest value of $f(x)$," the answer "$f(3)$" is incomplete; you would also need to evaluate f at 3).

Problem 1. Compute the derivatives of the following functions.

(a) $f(x) = 2x^3 - x + 7$

(b) $f(x) = \frac{1}{x}(x^5 - 2x^3 + 2)$

(c) $f(x) = \frac{2x-1}{x-2}$

The above problem is directed towards Objectives 4 and 6 (derivatives and rules of differentiation).

Problem 2. Let $f(x)$ be any function whose derivative $f'(x)$ exists for every x. Find the derivative of $f(x)^2$ in terms of $f(x)$ and $f'(x)$.

The above problem is primarily directed towards Objective 6 (rules of differentiation).

Problem 3. In class, we said that the power rule allows us to differentiate the square root function: $\frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{1/2} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$. However, the proof we gave of the power rule only applies to integer powers.

Show that nevertheless it is true that $\frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$ (for $x > 0$), using the definition of the derivative directly instead of the power rule.

The above problem is primarily directed towards Objective 4 (derivatives).

Challenge problem (1 point). Similarly to problem 3, we also said that the power rule applies to negative exponents, and you showed that it holds for $n = -1$ (i.e. that $\frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2}$)

on a worksheet. Use the binomial theorem and a similar method to show that in fact the power rule holds for all negative n.

Challenge problem (2 points). Use the product rule (*not* the quotient rule or the chain rule, if you know it) to find a formula for $\frac{d}{dx}\frac{1}{f(x)}$.

Survey. Estimate the amount of time you spent on each problem to the nearest half hour.

	Time Spent
Problem 1	
Problem 2	
Problem 3	
Challenge problem 1	
Challenge problem 2	