

## Calculus I, section 10: examples of standards

### Satisfactory

**Satisfactory**, or S, work completely and correctly solves the given problem, and is clearly and legibly explained. There may be at most one or two very minor errors such as typos or misspellings, but nothing that significantly affects that mathematical content.

For example, suppose the problem is the following (say we are learning how to solve quadratic equations):

**Problem.** Find all *positive* solutions to  $2x^2 + x = 3$ .

An example of a satisfactory solution is as follows:

**Solution.** Moving the 3 to the other side of the equation, this is  $2x^2 + x - 3 = 0$ . Now we can factor the left-hand side:  $(x - 1)(2x + 3) = 0$ . Since the product of two numbers can only be zero if at least one of them is zero, we have either  $x - 1 = 0$  or  $2x + 3 = 0$ , i.e. (adding 1 to both sides)  $x = 1$  or (subtracting 3 and dividing by 2)  $x = -\frac{2}{3}$ . Since the problem asks for positive solutions, only  $x = 1$  works, and so the answer is just  $x = 1$ .

This answer is completely correct, and it is (reasonably) easy to follow what is being done.

Another example of a satisfactory solution, which in this case contains very minor flaws, is as follows.

**Solution.** Moving the 3 to the other side of the equation, this is  $2x^2 + x - 3 = 0$ . Now we can apply the quartic formula:  $x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot (-3)}}{2 \cdot 2} = \frac{-1 \pm \sqrt{25}}{4}$ , which is either  $-\frac{2}{3}$  or 1. Since 1 is positive it is the only correct answer.

This solution gets the right answer, and the explanation is reasonably clear. However there is less explanation than in the previous example, and if there were a significant error this would make it harder to find. In addition, the student has mistakenly written “quartic” instead of “quadratic,” and the conclusion that 1 is the only correct answer because it is positive is not, strictly speaking, correct; it is also necessary to check that  $-\frac{2}{3}$  is negative. However, it’s clear that this student understands what is going on and how to solve the problem, and no error is really significant.

### Minor revisions

Work is graded **minor revisions**, or M, if it is largely correct, but either has significant minor errors or displays an understanding of the underlying concepts which is not completely correct. Work which is mathematically completely correct may also be graded as M if it is written unclearly and the grader cannot be certain that the solution is completely correct.

Let's use the same example problem as above. Here is an example of a solution which would receive the grade M.

**Solution.** This is the same thing as  $2x^2 + x - 3 = 0$ , which by the quadratic formula is  $x = \frac{-1 \pm \sqrt{23}}{4}$ . Since  $\sqrt{23} > 1$ , the only positive solution is  $x = \frac{\sqrt{23}-1}{4}$ .

This student has an essentially correct understanding, but has been careless: by skipping a few steps, they have accidentally flipped a sign and so gotten the wrong answer. This is a minor mistake but does meaningfully affect the math involved; revising this solution to make it satisfactory would be straightforward. A similar example would be if the student did the calculation correctly, but forgot to restrict to positive solutions.

A more problematic example is as follows.

**Solution.** This is the same thing as  $2x^2 + x - 3 = 0$ , which we can factor as  $(x-1)(2x+3)$ , so either  $x = -1$  or  $x = \frac{2}{3}$ . Only the second one is positive, so the answer is just  $x = \frac{2}{3}$ .

This student has correctly set up the equation and factored, but they do not completely understand the factoring method: they know that the solution is somehow related to the factors, but not why and therefore have made a sign error on each term. This will take some studying and discussion to fully understand this concept, but they do have an understanding of the concept, if incomplete, and it is possible that the student has just made a straightforward sign error. The revision for this solution should include an explanation of how to get the correct solution from the factoring and why it works.

## Not yet satisfactory

Work is graded **not yet satisfactory** if it does not achieve either of the former two grades, in other words if it has major errors, exhibits a significantly incorrect understanding of the concepts, or is written in such a way that the grader cannot reasonably assess it. Here is an example with the same problem as above.

**Solution.** The left-hand side is  $x(2x+1)$ , so either  $x = 3$  or  $2x+1 = 3$ , i.e.  $x = 3$  or  $x = 1$ . Both are positive, so the solution is both.

This exhibits a major misunderstanding of factoring, and the student could have easily checked that the resulting answer is not correct. A reattempt would need to do the problem from scratch, explaining why the solution works.

Another example is:

**Solution.**  $x = 1$ .

Although strictly speaking the final answer is correct, there is nothing here to grade: any solution necessarily includes the method, and this has none.