Project guidelines

Calculus I, section 10

1. Timeline and requirements

You may (optionally!) choose to replace the third midterm exam with a project. The goal of each project is to 1) investigate some problem using the mathematical concepts we've studied in this class, 2) write an expository paper on the topic, i.e. explain it in detail to an audience unfamiliar with it, and 3) give a short presentation to the class on your topic at the end of the semester. Depending on the project, your exposition may also involve computer simulations, in which case the expository paper might (if appropriate) be correspondingly shorter but should explain these simulations.

This is a fairly open-ended project; you may use any resources you like, and the converse of this is that it is your job to find (and properly cite) references to understand your desired topic. That said, I am happy to help you find resources if you are having difficulty, especially on more obscure topics.

Timeline

The deadlines are as follows, all by class time, i.e. 4:10 PM.

November 1 Deadline to let me know via email if you would like to do a project, with your choice of topic (see below).

November 10 Deadline to submit an outline (at most one page) of what you plan to do and what sources you intend to use, including examples and exercises. This should be thought of as a rolling deadline: the sooner you submit your outline, the sooner you will get feedback on it, which will give you more time to incorporate this feedback into your final draft.

November 15 Deadline to submit a final draft of your project. This is what you will be graded on; however like all assignments in this class you can continue to submit further edits incorporating feedback to improve your mark until the end of the class. A project resubmission will count towards the weekly cap as two reattempts.

Guidelines

The primary goal of this project is to understand the mathematics of your topic; nearly as important however is clearly communicating that understanding. Imagine that you are trying to explain the concepts you have studied to someone who took a calculus 1 course some years in the past and so vaguely recalls the material but may need you to review the more complicated topics.

Like any paper, in addition to the main body of the exposition your paper should include a short introduction, explaining the main ideas, motivation, and background of your paper, as well as a list of sources. (The specific formatting of your sources does not matter so long as it is clear.)

All papers should be typed.[1] I encourage you to use LaTeX[2], and am happy to hold workshops on it if there is interest; however it is not required, and you may use whatever software you prefer.

There is no hard guideline for the length of your papers: they should be the length they need to be in order to concisely and clearly explain your topic in detail to the appropriate audience, but in practice probably papers should be at least around two pages (possibly less, but only if they are very well-written).

Grading

Your paper should be clearly targeted at at least one of the objectives for this unit, namely extrema, optimization, L'Hôpital's rule, or various other applications of derivatives, and will be counted towards whichever of the objectives it is targeted at (as well as potentially other earlier objectives it touches on). In other words, you should think of the choice to do a project, rather than an exam, as sacrificing some breadth in order to go into greater depth on one or two topics. Unlike other assignments, for the project you'll receive one mark (S, M, or N as usual) for each relevant objective. What I'm looking for in these projects (together with the required steps, such as submitting an outline and the project on time) is as follows, roughly in order of importance:

- mathematical correctness;
- clarity and quality of exposition;
- depth, style, creativity.

2. Topic suggestions

Any of these should be taken as a collection of related possible ideas around which to base your project; you do not necessarily need to cover everything mentioned, and might cover aspects not mentioned.

Newton's method and extrema. In order to apply our method to find extrema, we need to be able to solve the equation $f'(x) = 0$. Sometimes, this is possible analytically, but sometimes it is not: for example, if $f(x) = e^x - x^2$, then $f(x)$ has a local minimum at about $x \approx 2.153$ and a local maximum at $x \approx 0.357$, but neither has an exact expression. Explain how to use Newton's method together with our method from class to numerically find maxima or minima, and give some illustrative examples.

[1] If this is a particular hardship for you, we can discuss alternatives.

[2] LaTeX is a software system for creating documents, especially those involving large numbers of mathematical or scientific symbols, and is probably what virtually all mathematical documents you have encountered at least in college were written in, including this one; there are many editors available, including online ones such as `overleaf.com`.

Second-order approximation. We talked about how to use derivatives to give first-order approximations to functions, both near a fixed point and to describe how a function changes near a very large input (e.g. to describe $\sqrt{x+1}$ in terms of $\sqrt{x}$ when x is large). Discuss how you could extend these ideas to a second-order approximation, using the second derivative, and give some examples. If you feel so moved, you could even look at higher-order approximations.

Optimization of non-smooth functions. Our method for optimization requires that we be able to differentiate the function in question. We saw how to deal with situations where there are a few points at which the function is not differentiable, but what if there are many? (What if it's *nowhere* differentiable?) Look into other approaches that could be used, possibly in conjunction with our method, to at least approximate the answer to an optimization problem, and give some examples.

L'Hôpital's rule for polynomials and polynomial approximation. We saw in class why L'Hôpital's rule is true in general. In a different way, work out specifically for polynomials why it and its iterations are true. Via these ideas, one can view L'Hôpital's rule as saying that near a given point, a function is "approximated" by polynomials; explain this.

In-depth optimization problems. Choose an optimization problem from a field of your choice, perhaps one you've encountered in other contexts. Model it using the tools from this unit, and solve the resulting optimization problem. Does this agree with your expectations or experience?

(Note that since everything depends on the choice of problem here, make sure you don't choose too easy or too hard a problem—it should be hard enough that a detailed explanation and solution is of a sufficient scope for the project, but not so hard that you can't solve it. If you're unsure, talk to me about it.)

Choose your own. Propose your own topic! It should be based on the material from this class and in particular this unit on applications of derivatives. Otherwise you are free to choose any topic that interests you, using the above as a guide.