Practice problems for final exam

Calculus I, section 10
December 13, 2022

These are practice problems for the content of the final exam. Full written solutions will be posted by Thursday night for your use in studying. I encourage you to attempt them prior to that on your own.

Problem 1. If $f(x) = \ln(x)$ and $g(x) = x^2 + 1$, find x such that $f(g(x)) = 0$.

The above problem is primarily directed towards Objective 1 (functions and algebra).

Problem 2. Find

$$\lim_{x \to 1} \frac{x^2 - 2x + 1}{x^2 + 2x - 3}.$$

The above problem is primarily directed towards Objective 2 (limits).

Problem 3. Let

$$f(x) = \begin{cases} \frac{1}{2}x + 1 & x > 0 \\ 1 - x & x < 0 \\ 0 & x = 0 \end{cases}.$$

Find and classify the discontinuities of $f(x)$.

The above problem is primarily directed towards Objective 3 (continuity).

Problem 4. Find the horizontal, vertical, and diagonal asymptotes (if they exist) of the function $f(x) = e^{1/x}$.

The above problem is primarily directed towards Objective 4 (asymptotes).

Problem 5. Directly using the definition of the derivative (i.e. *not* using the power or quotient rules or similar), find the derivative of $f(x) = \frac{1}{x}$.

The above problem is primarily directed towards Objective 5 (derivatives).

Problem 6. Let c be a real number, and let

$$f(x) = \begin{cases} x^3 & x > 1 \\ cx - c + 1 & x \leq 1 \end{cases}.$$

Find the value of c that makes $f(x)$ differentiable everywhere, if it exists.

The above problem is primarily directed towards Objective 6 (differentiability).

Problem 7. Find the derivatives of

(a) $x^2 \sin(x)$, and

(b) $\sin(x)^2$.

The above problem is primarily directed towards Objective 7 (rules of differentiation).

Problem 8. Find the derivative of $\frac{1}{x}\ln(1+\sqrt{x})$.

The above problem is primarily directed towards Objective 8 (combining rules of differentiation).

Problem 9. Find and classify the critical points of $\sin(x)+\cos(x)$ between 0 and 2π.

The above problem is primarily directed towards Objective 9 (extrema).

Problem 10. Given 40 feet of fencing, you can make a rectangle of dimensions $x \times y$, with perimeter $2x+2y=40$. Find the values of x and y which give the greatest area contained inside this rectangle.

The above problem is primarily directed towards Objective 10 (optimization).

Problem 11. Find

$$\lim_{x\to -1}\frac{\sin(\pi x)}{x+1}.$$

The above problem is primarily directed towards Objective 11 (L'Hôpital's rule).

Problem 12. If $x(t)y(t)=4$, $x(1)=2$, and $x'(1)=-1$, find $y'(1)$.

The above problem is primarily directed towards Objective 12 (assorted applications of differentiation).

Problem 13.

(a) Compute $\displaystyle\int_0^1 x\,dx$.

(b) Find an antiderivative of $2x^3-1$.

The above problem is primarily directed towards Objective 13 (integrals).

Problem 14. Evaluate $\displaystyle\int_{-\pi}^{\pi} 2\sin(x)\,dx$.

The above problem is primarily directed towards Objective 14 (the fundamental theorem of calculus).

Problem 15. Find $\displaystyle\int x\sqrt{x^2+1}\,dx$.

The above problem is primarily directed towards Objective 15 (u-substitution).

Problem 16. Find the average value of $f(x)=e^{x-1}$ between -1 and 1.

The above problem is primarily directed towards Objective 16 (applications of integrals).