Practice problems for midterm 3

Calculus I, section 10

November 12

These are practice problems for the content of the first midterm. This is *not* a practice test, and you should not expect it to necessarily approximate the test in either length or difficulty; the problems on the test will likely be shorter and easier, at least on average. However, if you know the material well enough to be able to solve these problems, you are well-prepared for the midterm.

Written solutions are below. Some are worked out in more detail, while others are more brief; you should try and follow the logic in either case, and if you can't, ask either me (via email) or someone in the help room. We'll also go over some of them in class, at your request.

Problem 1. Find and classify the critical points of $f(x) = \ln(x) - \frac{2}{3}x + \frac{x^2}{24}$ (as local maxima, local minima, or inflection points) for $x > 0$.

The above problem is primarily directed towards Objective 9 (extrema).

Problem 2. Consider the following statement: "if $f(x)$ is a differentiable function with local minima at a and b, then f must have a local maximum somewhere between a and b." Is this statement true or false? Why?

The above problem is directed towards Objectives 9 and 12 (extrema and assorted applications of differentiation).

Problem 3. Evaluate

$$\lim_{x\to 0} \frac{x^2}{e^x - x - 1}$$

using L'Hôpital's rule.

The above problem is primarily directed towards Objective 11 (L'Hôpital's rule).

Problem 4. The *entropy* H of a probability $0 \le p \le 1$ is defined to be

$$H(p) = p \cdot \log_2\left(\frac{1}{p}\right) + (1-p)\cdot \log_2\left(\frac{1}{1-p}\right).$$

Find the probability which makes the entropy largest, i.e. the p which maximizes $H(p)$. You may use without proof the fact that $\lim_{x\to 0} x\log_2(x) = 0$.

The above problem is primarily directed towards Objective 10 (optimization).

Problem 5. Suppose we're trying to use Newton's method to find $\ln(2)$, i.e. the unique solution to $e^x - 2 = 0$. Show that if we choose x_0 very large, then $x_1 < x_0$, and similarly if we choose x_0 much less than 0 (approaching $-\infty$) then $x_1 > x_0$. (This suggests Newton's method might converge in this case.)

The above problem is primarily directed towards Objective 12 (assorted applications of differentiation).