Practice problems for midterm 2

Calculus I, section 10

October 16

These are practice problems for the content of the first midterm. This is *not* a practice test, and you should not expect it to necessarily approximate the test in either length or difficulty; the problems on the test will likely be shorter and easier, at least on average, and there will be fewer of them (4 - 5 instead of 7). However, if you know the material well enough to be able to solve these problems, you are well-prepared for the midterm.

Full written solutions will be posted by Tuesday night for your use in studying. I encourage you to attempt them prior to that on your own.

Problem 1. Suppose $f(x) = x^5$. We know that $f(2) = 2^5 = 32$. Using the derivative of $f(x)$ at $x = 2$, give an approximation to $f(2.01)$.
The above problem is directed towards Objectives 5 and 7 (derivatives and rules of differentiation).

Problem 2. Let $f(x) = x^{\frac{1}{x+1}}$ for $x \geq 0$. Find a formula for $f'(x)$. For which $x \geq 0$ is $f(x)$ differentiable?
The above problem is directed towards Objectives 6 and 7 (differentiability and rules of differentiation).

Problem 3. If $\sin(y) = x\cos(x)$, find $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$, $y = 0$.
The above problem is directed towards Objectives 7 and 8 (rules of differentiation and combining rules).

Problem 4. Consider
$$f(x) = \begin{cases} 2 & x \geq 5 \\ 1 & 1 < x < 5 \\ x & x \leq 1 \end{cases}.$$

Find $f'(x)$ wherever it is differentiable, and determine where it is not differentiable.
The above problem is directed towards Objectives 5 and 6 (derivatives and differentiability).

Problem 5. Differentiate $f(x) = \frac{3^{-x}}{x^2}$.
The above problem is directed towards Objectives 7 and 8 (rules of differentiation and combining rules).

Problem 6. Let $f(x) = x^2 \sin^{-1}(x)$. Find $f'(0)$.
The above problem is primarily directed towards Objective 8 (combining rules of differentiation).