

Practice problems for midterm 1

Calculus I, section 10

September 24

These are practice problems for the content of the first midterm. This is *not* a practice test, and you should not expect it to necessarily approximate the test in either length or difficulty; the problems on the test will likely be shorter and easier, at least on average, and there will be fewer of them (4 - 5 instead of 7). However, if you know the material well enough to be able to solve these problems, you are well-prepared for the midterm.

Full written solutions will be posted by Thursday for your use in studying. I encourage you to attempt them prior to that on your own.

Problem 1. If $f(x) = \log_4(x)$, find $\lim_{x \rightarrow 0^+} 4^{f(x)}$, if it exists.

The above problem is directed towards Objectives 1 and 2 (functions and limits).

Problem 2. Let $f(x) = \frac{\cos(x)}{x}$. Find all horizontal, vertical, or diagonal asymptotes of $y = f(x)$.

The above problem is primarily directed towards Objective 4 (asymptotes).

Problem 3. Let

$$f(x) = \begin{cases} 2x & x > 1 \\ x^2 + 1 & x < 1 \end{cases}.$$

(a) Find all real numbers a where $f(x)$ has discontinuities, if any exist.

(b) Find all real numbers a where $f(f(x))$ has discontinuities, if any exist.

The above problem is directed towards Objectives 1 and 3 (functions and continuity).

Problem 4. Recall that we showed in class that for x near 0, we have $\cos(x) \leq \frac{\sin(x)}{x} \leq 1$, and concluded by the squeeze theorem that since $\lim_{x \rightarrow 0} \cos(x) = \lim_{x \rightarrow 0} 1 = 1$, we must also have $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$. Use these inequalities together with the squeeze theorem to similarly show that $\lim_{x \rightarrow 0} \frac{\tan(x)}{x} = 1$.

The above problem is primarily directed towards Objective 2 (limits).

Problem 5. Evaluate the limit $\lim_{x \rightarrow 0} 3^{\frac{1}{x^2}}$, if it exists.

The above problem is primarily directed towards Objective 2 (limits).

Problem 6. Is it possible for there to be a function $f(x)$ such that $f(x) + \sin(x)$ has a horizontal asymptote? If so, give an example; if not, explain why not.

The above problem is primarily directed towards Objective 4 (asymptotes).

Problem 7. Consider the function $f(x) = \frac{\sqrt{|x|} - 1}{x^2 - 4x + 3}$. Find and classify all of its discontinuities, if any exist.

The above problem is primarily directed towards Objective 3 (continuity).