

# Practice problems for midterm 1

Calculus I, section 10

September 24

These are practice problems for the content of the first midterm. This is *not* a practice test, and you should not expect it to necessarily approximate the test in either length or difficulty; the problems on the test will likely be shorter and easier, at least on average, and there will be fewer of them (4 - 5 instead of 7). However, if you know the material well enough to be able to solve these problems, you are well-prepared for the midterm.

Full written solutions will be posted by Thursday for your use in studying. I encourage you to attempt them prior to that on your own.

**Problem 1.** If  $f(x) = \log_4(x)$ , find  $\lim_{x \rightarrow 0^+} 4^{f(x)}$ , if it exists.

*The above problem is directed towards Objectives 1 and 2 (functions and limits).*

**Problem 2.** Let  $f(x) = \frac{\cos(x)}{x}$ . Find all horizontal, vertical, or diagonal asymptotes of  $y = f(x)$ .

*The above problem is primarily directed towards Objective 4 (asymptotes).*

**Problem 3.** Let

$$f(x) = \begin{cases} 2x & x > 1 \\ x^2 + 1 & x < 1 \end{cases}.$$

(a) Find all real numbers  $a$  where  $f(x)$  has discontinuities, if any exist.

(b) Find all real numbers  $a$  where  $f(f(x))$  has discontinuities, if any exist.

*The above problem is directed towards Objectives 1 and 3 (functions and continuity).*

**Problem 4.** Recall that we showed in class that for  $x$  near 0, we have  $\cos(x) \leq \frac{\sin(x)}{x} \leq 1$ , and concluded by the squeeze theorem that since  $\lim_{x \rightarrow 0} \cos(x) = \lim_{x \rightarrow 0} 1 = 1$ , we must also have  $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$ . Use these inequalities together with the squeeze theorem to similarly show that  $\lim_{x \rightarrow 0} \frac{\tan(x)}{x} = 1$ .

*The above problem is primarily directed towards Objective 2 (limits).*

**Problem 5.** Evaluate the limit  $\lim_{x \rightarrow 0} 3^{\frac{1}{x^2}}$ , if it exists.

*The above problem is primarily directed towards Objective 2 (limits).*

**Problem 6.** Is it possible for there to be a function  $f(x)$  such that  $f(x) + \sin(x)$  has a horizontal asymptote? If so, give an example; if not, explain why not.

*The above problem is primarily directed towards Objective 4 (asymptotes).*

**Problem 7.** Consider the function  $f(x) = \frac{\sqrt{|x|}-1}{x^2-4x+3}$ . Find and classify all of its discontinuities, if any exist.

*The above problem is primarily directed towards Objective 3 (continuity).*