Homework 7

Calculus I, section 10

Due November 9, 2022 by 4:10 PM

As usual, you may use any resources to solve these problems except where stated otherwise, with the exception of computational software and posting these problems anywhere to be answered by others. Collaboration is encouraged, but everyone should write their own solutions. Write the names of any collaborators or sources used at the top of your homework. If you did not use any sources, write "sources used: none."

If you find any errors in either the homework or the lecture notes, please let me know, even if you are unsure whether it is an error or not.

As on most math problems, the mathematics is the issue, not the answer: whether you have a correct method is more important then whether you get to the correct number at the end, so include your method!

You do not have to simplify your answers completely (so for example $\frac{2}{2}$ is fine), but you do need to do all the computations (so for example if the problem is "find the largest value of $f(x)$," the answer "$f(3)$" is incomplete; you would also need to evaluate f at 3).

Problem 1. A truck driving at v miles per hour has a gas mileage of $\frac{1}{a+bv^2}$ miles per gallon of gasoline, where a and b are some fixed positive constants (depending on the design and fuel efficiency of the truck). The truck driver is paid by the mile, at a rate of r dollars per mile, but has to pay for gasoline, which costs g dollars per gallon.

(a) Assume the driver drives at a constant speed v. Write down an expression $M(v)$ for the total net pay of the driver (i.e. how much they get paid minus gasoline costs) after one hour, in terms of the speed v and the constants a, b, r, and g.

(b) Using your expression from part (a), determine at which speed v the driver should drive in order to maximize their profit if $a = 0.04$ (in units of gallons), $b = 0.00001$ (in units of gallons times meters squared per seconds squared), $r = \$0.50$ per mile, and $g = \$4$ per gallon and the speed limit (which the driver does not exceed) is 70 miles per hour.

The above problem is primarily directed towards Objective 10 (optimization).

Problem 2. Evaluate the following limits using L'Hôpital's rule.

(a) $\lim_{x\to 0} \frac{\sin(x)-\tan(x)}{x^3}$

(b) $\lim_{x\to+\infty} \frac{x^3}{e^x}$.

The above problem is primarily directed towards Objective 11 (L'Hôpital's rule).

Problem 3. Let $f(x) = x\sin(x)$.

(a) Show that for $0 \le x \le 0.01$, we have $|f''(x)| \le 2$.

(b) Using the formula for the error in first-order approximation together with part (a), conclude that $|f(x)| \leq x^2$ for $0 \leq x \leq 0.01$.

The above problem is primarily directed towards Objective 12 (assorted applications of differentiation).

Survey. Complete the following survey by rating each problem you attempted on a scale of 1 to 10 according to how interesting you found it (1 = "mind-numbing," 10 = "mind-blowing"), and how difficult you found it (1 = "trivial," 10 = "brutal"). Also estimate the amount of time you spent on each problem to the nearest half hour.

	Interest	Difficulty	Time Spent
Problem 1			
Problem 2			
Problem 3			