Homework 2

Calculus I, section 10

Due September 20, 2022 by 4:10 PM

As usual, you may use any resources to solve these problems except where stated otherwise, with the exception of computational software and posting these problems anywhere to be answered by others. Collaboration is encouraged, but everyone should write their own solutions. Write the names of any collaborators or sources used at the top of your homework. If you did not use any sources, write "sources used: none."

If you find any errors in either the homework or the lecture notes, please let me know, even if you are unsure whether it is an error or not.

As on most math problems, the mathematics is the issue, not the answer: whether you have a correct method is more important then whether you get to the correct number at the end, so include your method!

You do not have to simplify your answers completely (so for example $\frac{2}{2}$ is fine), but you do need to do all the computations (so for example if the problem is "find the largest value of $f(x)$," the answer "$f(3)$" is incomplete; you would also need to evaluate f at 3).

Problem 1. Evaluate each of the following limits. (If it does not exist, say so.)

(a) $\lim_{x\to 0}\left(\frac{1}{x}+\frac{4}{x^2-4x}\right)$

(b) $\lim_{x\to 0^+}\frac{\sin(x)}{\sqrt{x}}$ (hint: use the fact from class that $\lim_{x\to 0}\frac{\sin(x)}{x}=1$).

Problem 2.

(a) Find $\lim_{x\to 0}\cos(x)$. (The answer should be 1. This is not a trick question, don't spend too much time on it.)

(b) Show that $\lim_{x\to 0}\frac{\cos(x)-1}{x}=0$. (Hint: use the squeeze theorem, together with a suitable trigonometric identity and the fact we proved in class, $|\sin(x)|\leq|x|$.)

(c) (BONUS PROBLEM: ungraded) Conclude that for x close to 0, $\cos(x)$ is approximated by $1-\frac{x^2}{2}$.

Problem 3. Is the following solution correct? If not, identify the mistake and explain what the solver should have done.

> **Problem.** Evaluate $\lim_{x\to\infty} x^{\frac{1}{\log_2(x)}}$.
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> **Solution.** By the power limit law (following from the multiplication limit law), this is the same thing as first evaluating $\lim_{x\to\infty}\frac{1}{\log_2(x)}$, and then plugging the result into the first limit and evaluating. Since $\log_2(x)$ goes to infinity as $x\to\infty$, $\frac{1}{\log_2(x)}\to 0$ as $x\to\infty$ and therefore the original limit is $\lim_{x\to\infty} x^0 = 1$.

Survey. Complete the following survey by rating each problem you attempted on a scale of 1 to 10 according to how interesting you found it (1 = "mind-numbing," 10 = "mind-blowing"), and how difficult you found it (1 = "trivial," 10 = "brutal"). Also estimate the amount of time you spent on each problem to the nearest half hour.

	Interest	Difficulty	Time Spent
Problem 1			
Problem 2			
Problem 3			