

# Homework 1

Calculus I, section 10

Due September 13, 2022 by 4:10 PM

As usual, you may use any resources to solve these problems except where stated otherwise, with the exception of computational software and posting these problems anywhere to be answered by others. Collaboration is encouraged, but everyone should write their own solutions. Write the names of any collaborators or sources used at the top of your homework. If you did not use any sources, write “sources used: none.”

If you find any errors in either the homework or the lecture notes, please let me know, even if you are unsure whether it is an error or not.

As on most math problems, the mathematics is the issue, not the answer: whether you have a correct method is more important than whether you get to the correct number at the end, so include your method!

You do not have to simplify your answers completely (so for example  $\frac{2}{2}$  is fine), but you do need to do all the computations (so for example if the problem is “find the largest value of  $f(x)$ ,” the answer “ $f(3)$ ” is incomplete; you would also need to evaluate  $f$  at 3).

**Problem 1.** Consider the function  $f(x) = x^3 - x^2 - 2x$ .

- (a) Find all real numbers  $x$  such that  $f(x) = 0$ .
- (b) Suppose that  $x$  is very large. Do you expect that  $f(x)$  is larger or smaller than  $x$ ? What about  $f(-x)$  vs.  $-x$ ?

**Problem 2.** Let  $\sin^{-1} : [-1, 1] \rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$  denote the inverse function of  $\sin : [-\frac{\pi}{2}, \frac{\pi}{2}] \rightarrow \mathbb{R}$ . Here  $[a, b]$  means the interval between  $a$  and  $b$ , i.e. the set of real numbers  $x$  such that  $a \leq x \leq b$ .

- (a) Show that  $\cos(\sin^{-1}(x)) = 1 - x^2$  for every  $x$  in the domain of  $\sin^{-1}$ .
- (b) Show that  $\sin^{-1}(\cos(x)) = \frac{\pi}{2} - |x|$  for every  $x$  in the domain of  $\sin^{-1}$ .

**Problem 3.** If  $y = 2^x$  and  $z = 3^y$ , find the value of  $x$  such that  $z = 2$ .

**Problem 4.** Consider the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 2^{x^3} - 1$ .

- (a) Is  $f(x)$  invertible? If so, give a formula for its inverse function; if not, explain why not.
- (b) If  $g = f \circ f \circ f$ , give a formula for  $f^{-1} \circ g \circ f^{-1}$ .

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**Survey.** Complete the following survey by rating each problem you attempted on a scale of 1 to 10 according to how interesting you found it (1 = “mind-numbing,” 10 = “mind-blowing”), and how difficult you found it (1 = “trivial,” 10 = “brutal”). Also estimate the amount of time you spent on each problem to the nearest half hour.

	Interest	Difficulty	Time Spent
Problem 1			
Problem 2			
Problem 3			
Problem 4			