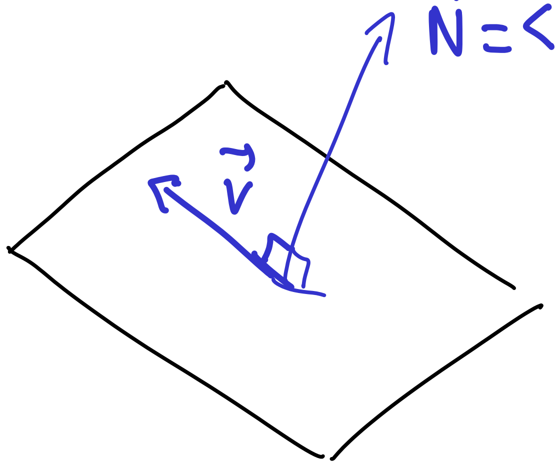
The vector $\vec{v} = \langle 1,2,-1 \rangle$ and the plane $x+y+3z=5$ are

1. parallel
2. ~~perpendicular~~
3. ~~neither~~



$\vec{N} = \langle 1,1,3 \rangle$

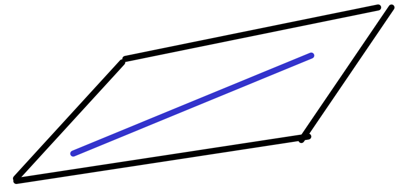
$\vec{v}$ not $\parallel \vec{N}$

$$\vec{v} \cdot \vec{N} = \langle 1,2,-1 \rangle \cdot \langle 1,1,3 \rangle$$
$$= 1+2-3 = 0$$

$\vec{v} \perp \vec{N}$

A LINE AND A PLANE:

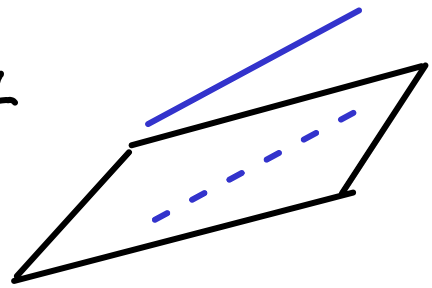
→ The line can be contained in the plane



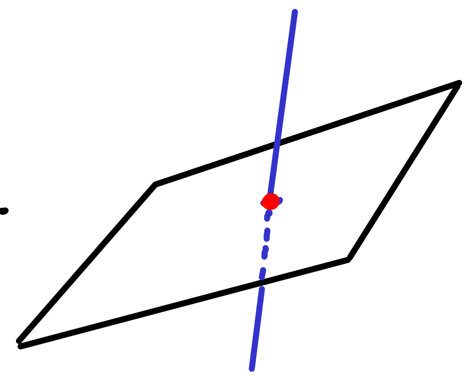
$\vec{v} \cdot \vec{N} = 0$

(in line) (normal)

→ or parallel to it



→ or intersects the plane in a point



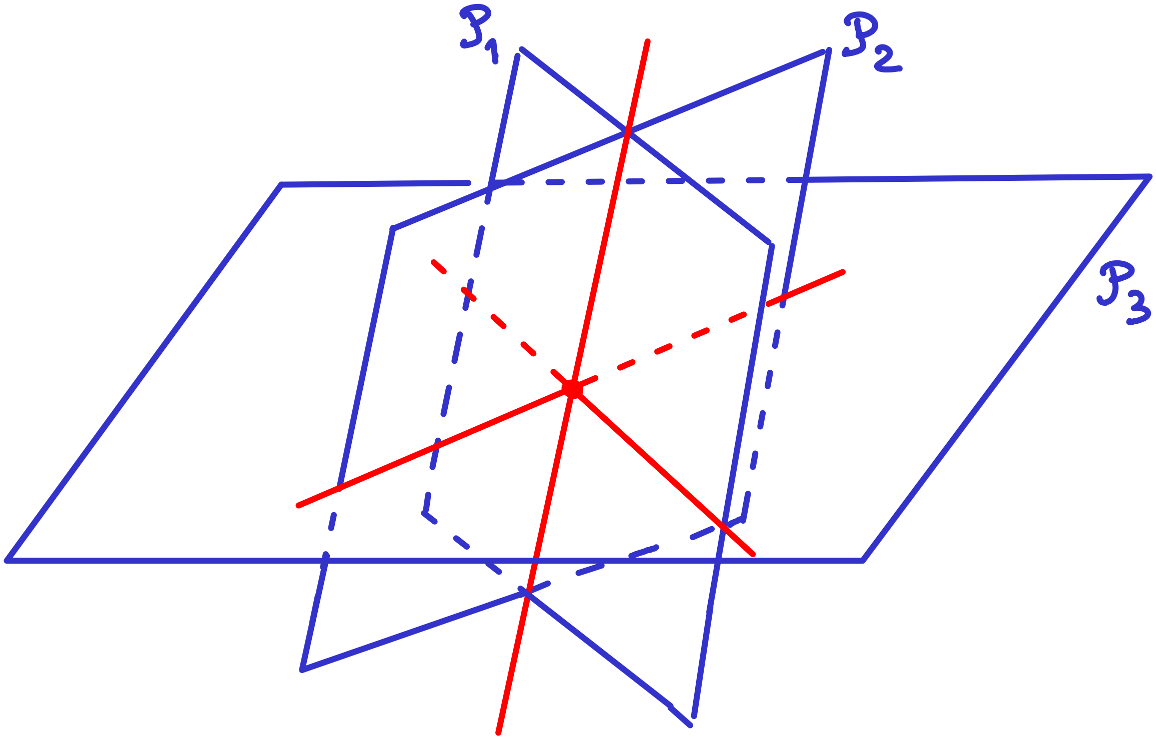
This has to do with: 3×3 linear systems!

$$\begin{cases} x+y+2z=7 & (\mathcal{P}_1) \\ 2x+y-z=4 & (\mathcal{P}_2) \\ x+2y+3z=3 & (\mathcal{P}_3) \end{cases}$$

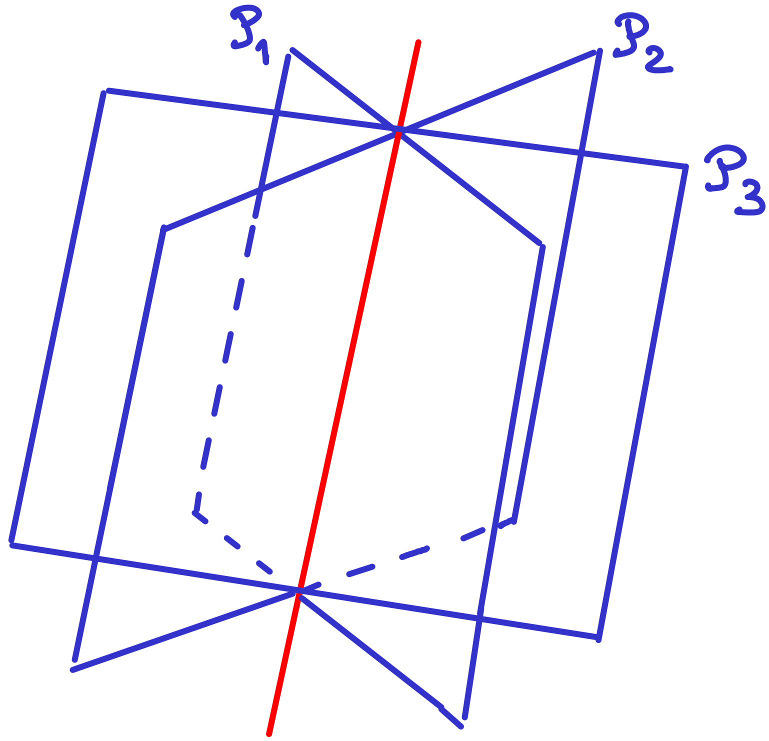
3 planes: where do they intersect?

How many solutions?

If the first 2 planes are not parallel, they intersect in a <u>line</u>.

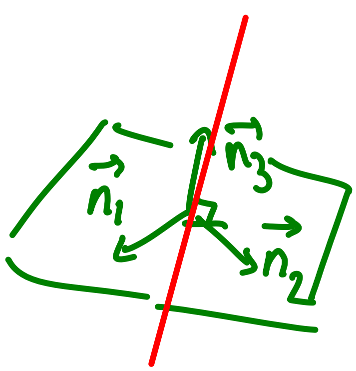


Line and P_3
intersect in a point
(1 solution)



Line $/\!/ \mathcal{P}_3$
(no solution)

Line contained in $\mathcal{P}_3$
(∞ solutions)

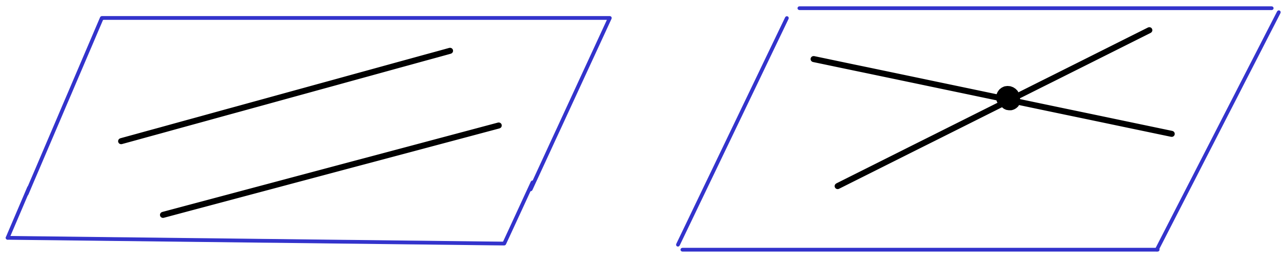


(Note: then the normal vectors to the 3 planes are all $\perp$ the line, hence coplanar.)

RELATIVE POSITIONS OF 2 LINES:

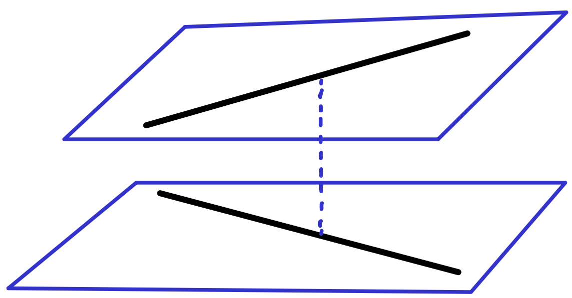
→ parallel
→ intersecting

} in both cases, a unique plane containing both lines



→ **skew lines** = neither parallel nor intersecting

Then they lie on parallel planes



Distance between the 2 planes = shortest distance between points on the skew lines.