Math 53 Homework 6

Due Wednesday 3/2/16 in section

(The problems in parentheses are for extra practice and optional. Only turn in the underlined problems.)

Monday 2/22 – Gradient, directional derivatives

- **Read:** section 14.6.
- **Work:** 14.6: 1, (7), 9, (11), (21), 27, (37), 38, 41, (43), 49, (51), 56.[1]
  Problems 1 and 2 below.

Wednesday 2/24 – Max-min problems

Friday 2/26 – Max-min problems continued

- **Read:** section 14.7.
- **Work:** 14.7: (1), 3, (7), 11, (23), (31), 36, (39), 43, (47), 49.[2]
  Problems 3 and 4 below.

Problem 1.

Find the maximum rate of change of $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at the point $(3, 6, -2)$, and the direction in which it occurs. Optional: find a geometric interpretation to your answer.

Problem 2.

a) Find the direction from $(4, 2, 3)$ in which $g(x, y, z) = x^2 + y^2 - 6z$ decreases fastest.

b) Follow the line in the direction you found in part (a) to estimate, using linear approximation, the location of the point closest to $(4, 2, 3)$ at which $g = 0$. Do not use a calculator. Express your answer using fractions. Next, use a calculator to evaluate g at your point. (The value should be reasonably small.)

(Hint: in the linear approximation, the closest point where $g = 0$ lies on the line through $(4, 2, 3)$ in the direction found in (a). Find a parametric equation for this line, then use linear approximation to estimate the value of g at a point on the line.)

Problem 3.

Find the absolute maximum and minimum values of $f(x, y) = x^4 + y^4 - 4xy + 2$ in the region $D = \{(x, y) \,|\, 0 \le x \le 3,\ 0 \le y \le 2\}$.

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[1] **6th ed:** 14.6: 1, (7), 9*, (11), (21), 27, (37), 38, 39, (41), 47, (49), 54. **7th ed** same as 8th.
*14.6 # 9: do the 7th/8th ed problem: $f(x, y, z) = x^2yz - xyz^3$, $P(2, -1, 1)$, $u = \langle 0, \frac{4}{5}, -\frac{3}{5} \rangle$.

[2] **6th and 7th eds:** 14.7: (1), 3, (7), 11*, (21), (29), 34, (37), 41, (45), 47.
*14.7 # 11: do the 8th ed problem: $f(x, y) = x^3 - 3x + 3xy^2$.

Problem 4.

Consider a triangle in the plane, with angles α, β, γ. Assume that the radius of its incircle is equal to 1.

a) By decomposing the triangle into six right triangles having the incenter as a common vertex, express the area A of the triangle in terms of α, β, γ (your answer should be a symmetric expression). Then use your result to show that A can be expressed as a function of the two variables α and β by the formula

$$A = \cot\frac{\alpha}{2} + \cot\frac{\beta}{2} + \tan\frac{\alpha+\beta}{2}.$$

b) What is the set of possible values for α and β? Find all the critical points of the function A in this region.

c) By computing the values of A at the critical points and its behavior on the boundary of the region where it is defined, find the maximum and the minimum of A (justify your answer). Describe the shapes of the triangles corresponding to these two situations.