Math 53 Homework 5

Due Wednesday 2/24/16 in section

(The problems in parentheses are for extra practice and optional. Only turn in the underlined problems.)

Wednesday 2/17 – Tangent plane, linear approximation

- **Read:** section 14.4†.

† PLEASE *don't mix differentials like* dz *with numerical differences like* Δx *or* Δy. *Statements such as "*$dx = \Delta x$*" do not make mathematical sense. See lecture.*

- **Work:** 14.4: (1), 5[1], 18[2], (19), 21, (25), 28, 33†, 38†, (39).

  Problem 1 below.

† *Whenever the book says "use differentials to estimate ...", read "use linear approximation to estimate ...".*

Friday 2/19 – Chain rule

- **Read:** section 14.5.
- **Work:** 14.5: (1), 5, 7[3], (13), 15, (17), 18[4], (23), (39), 43, 45, (47), 51, (53).

  Problems 2 and 3 below.

Problem 1.

Assume that $b^2 - 4c > 0$ so that $x^2 + bx + c = 0$ has two roots. Let r denote the larger root. Then r is a function of b and c.

a) Give an approximate formula for the small change Δr in the value of r produced by small changes Δb and Δc in the coefficients. Use this to calculate an approximate value for the larger root of $x^2 - 5.01x + 3.98 = 0$. Compare your answer with the exact value.

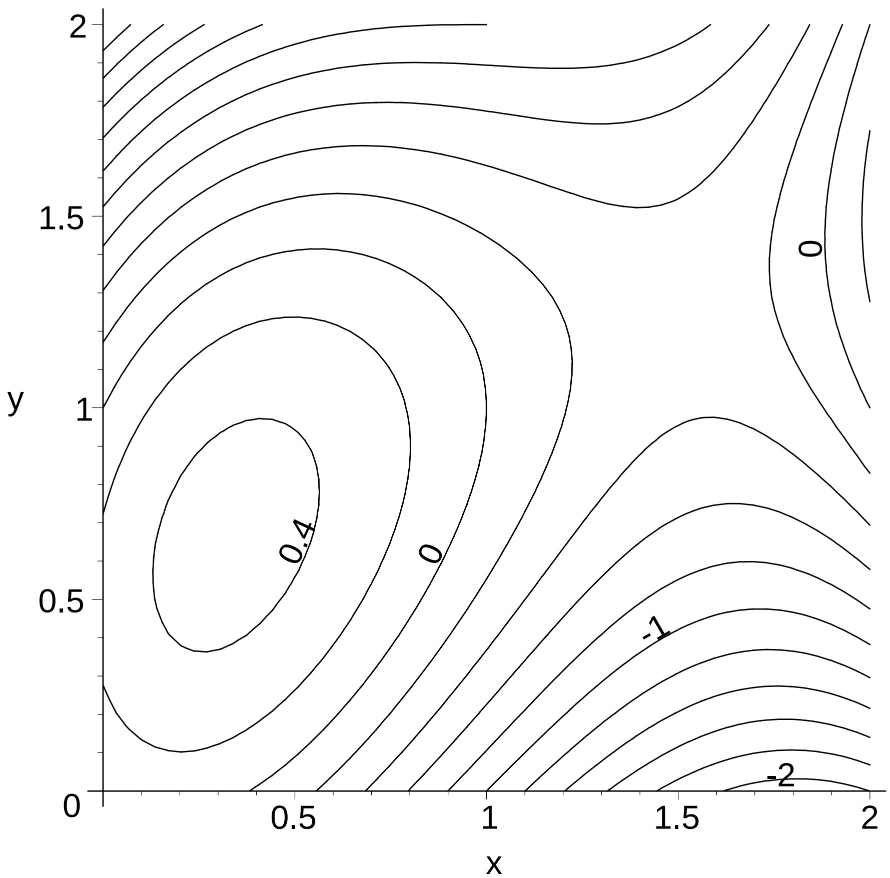
b) Starting from the equation $x^2 - 5x + 4 = 0$, is r more sensitive to small changes in b or c? (Justify your answer.)

Problem 2.

The figure on the next page is the contour plot of a function of two variables $f(x, y)$, for x and y ranging between 0 to 2 (scale: 1 unit = 5 cm; spacing between contour levels: 0.2).

a) Use the contour plot to determine whether f_x and f_y are > 0, $= 0$, or < 0 at the point $(\frac{3}{2}, \frac{1}{2})$. Same question at the point $(1, 1)$?

[1] **6th ed:** do the 7th/8th ed version, $z = x\sin(x + y)$, $(-1, 1, 0)$.
[2] **6th/7th eds:** do the 8th ed version, $(y - 1)/(x + 1) \approx x + y - 1$.
[3] **6th/7th eds:** do the 8th ed version: $z = (x - y)^5$, $x = s^2t$, $y = st^2$.
[4] **6th/7th eds:** do the 8th ed version: $w = f(x, y, z)$, where $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$.



b) Imagine that we move from the point $(\frac{3}{2}, \frac{1}{2})$ along a straight line whose direction is given by a unit vector $\langle a, b \rangle$, so $x(t) = \frac{3}{2} + at$ and $y(t) = \frac{1}{2} + bt$, and consider the value of $f(x(t), y(t))$ as a function of t. What are the possible direction(s) $\langle a, b \rangle$ for which $df/dt = 0$ at $t = 0$? (Either sketch a picture showing a portion of level curve and the vector $\langle a, b \rangle$, or give approximate angle from the x-axis). For each of these directions, when travelling along the chosen line, does the value of f pass through a minimum or a minimum at $t = 0$?

c) Use the contour plot to find two points where $f_x = f_y = 0$, and give their approximate coordinates. What happens to the level curves of f through these points? For each of the two points, describe what happens when you move towards North, South, East, West: does the value of f go up, down, or does it stay exactly the same?

Problem 3.

The function in the previous problem is $f(x, y) = x(x-1)(x-2) + (y-1)(x-y)$.

a) Calculate the actual values of the partial derivatives at $(\frac{3}{2}, \frac{1}{2})$ and $(1, 1)$.

b) At $(\frac{3}{2}, \frac{1}{2})$, find the unit vectors $\langle a, b \rangle$ such that the derivative of f along the line $x(t) = \frac{3}{2} + at$, $y(t) = \frac{1}{2} + bt$ is zero.

c) Find the points where $f_x = f_y = 0$, and calculate the second partial derivatives f_{xx} and f_{yy} at these points. Relate your answer to your findings in Problem 2(c).